



# Geomechanics

LECTURE 6

## CRITICAL STATE CONCEPT

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PROF. LYESSE LALOU

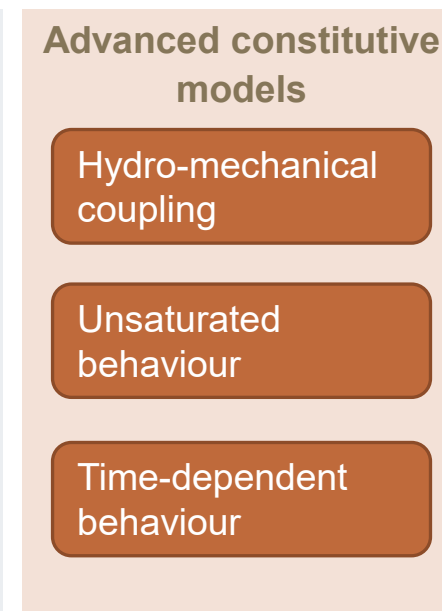
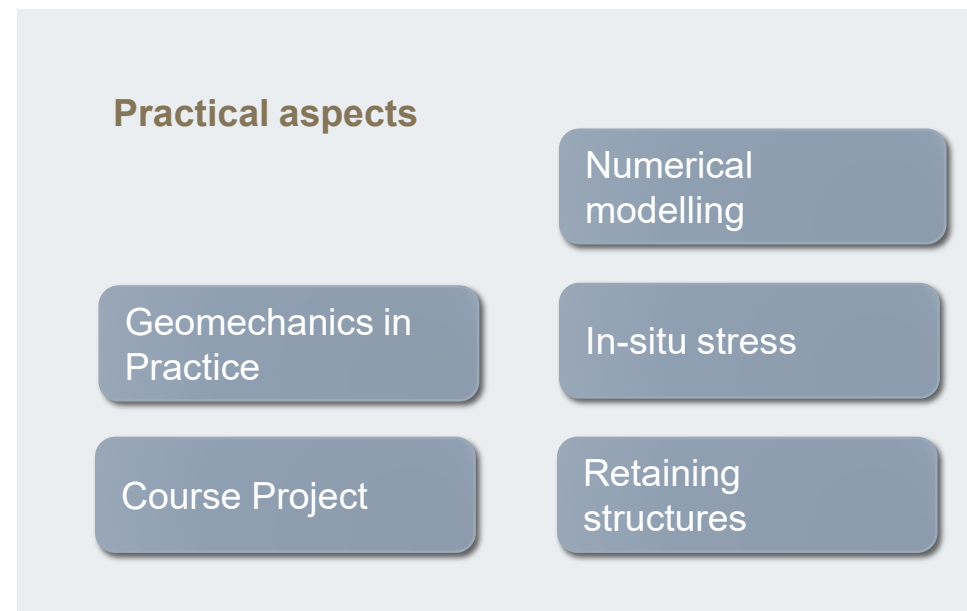
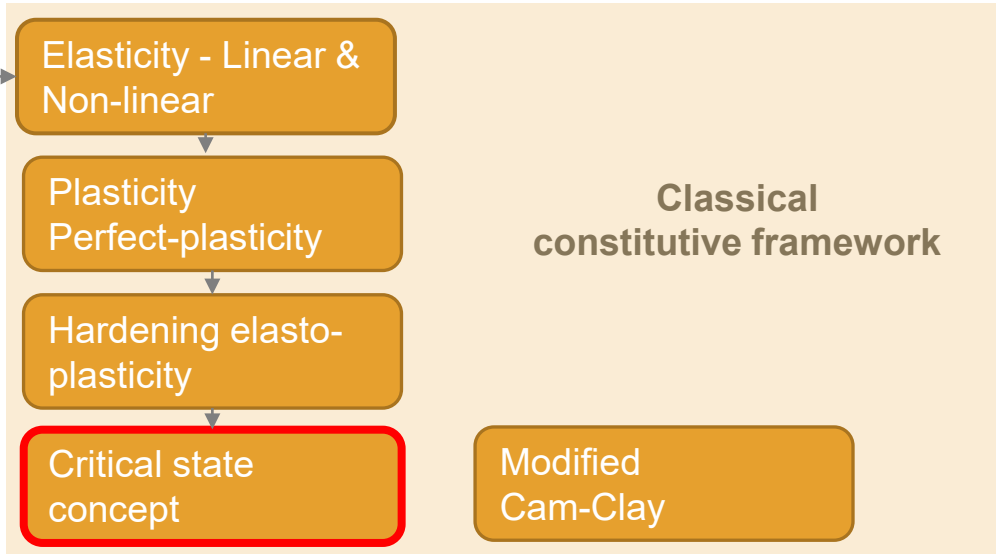
Laboratory of soil mechanics - Fall 2025

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Basic concepts



Topics

- Introduction
- Experimental evidences Conventional Triaxial Compression tests
  - Isotropic drained compression
  - Shearing phase on a NC soil in drained and undrained conditions
  - Shearing phase on a OC soil in drained and undrained conditions
- Critical state and shear strength
- Examples

- Introduction
- Experimental evidences Conventional Triaxial Compression tests
  - Isotropic drained compression
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- Critical state and shear strength
- Examples

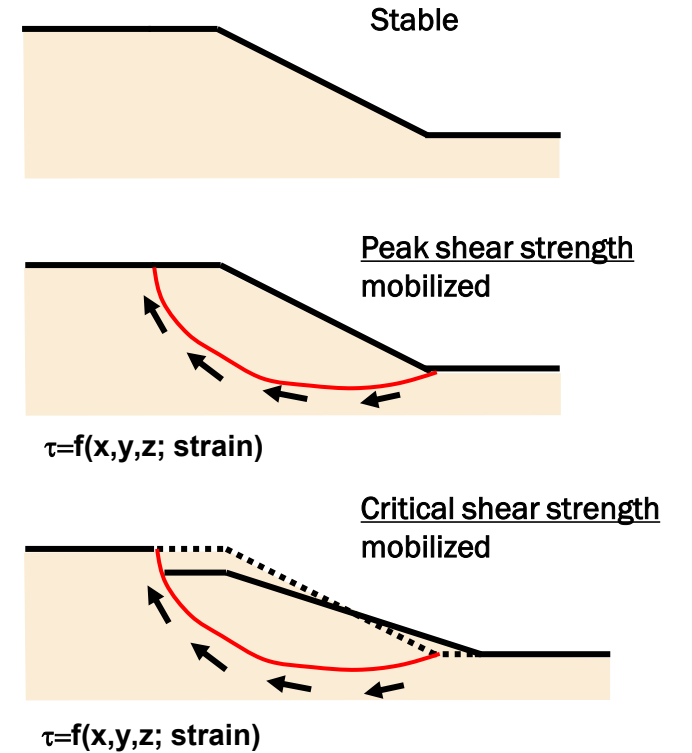
# The critical state concept

Practical example: progressive failure in a slope



Belmont/Lausanne landslide 1990

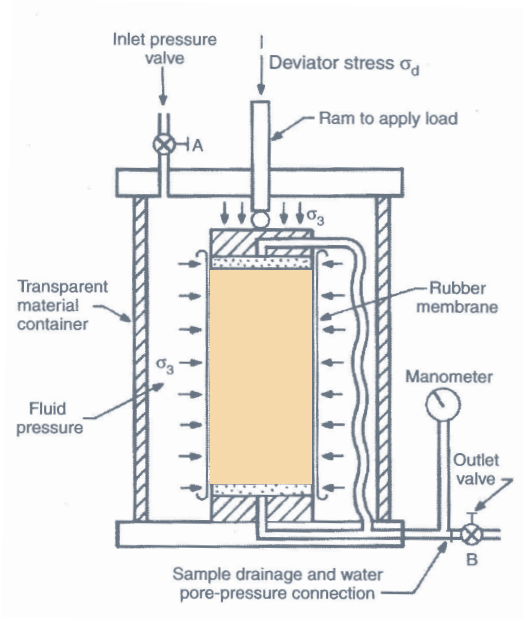
Critical shear strength mobilized (large strains)



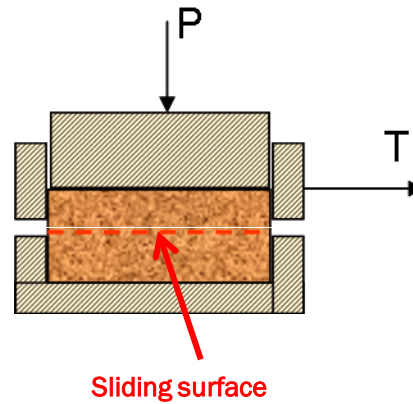
- Introduction
- Experimental evidences Conventional Triaxial Compression tests
  - **Isotropic drained** compression
  - Shearing phase on a NC soil in drained and undrained conditions
  - Shearing phase on a OC soil in drained and undrained conditions
- Critical state and shear strength
- Examples

# Experimental setups

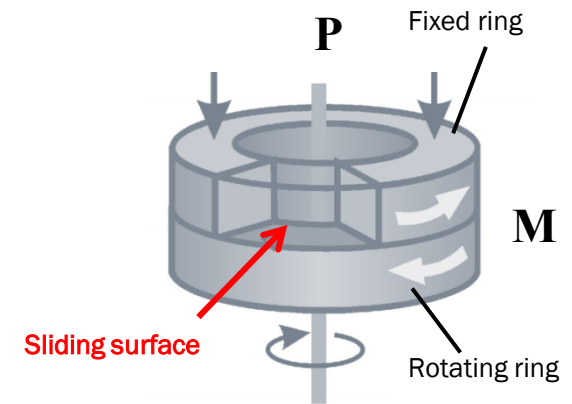
## Laboratory tests for analysing the shear strength of a soil



Triaxial test



Direct shear test



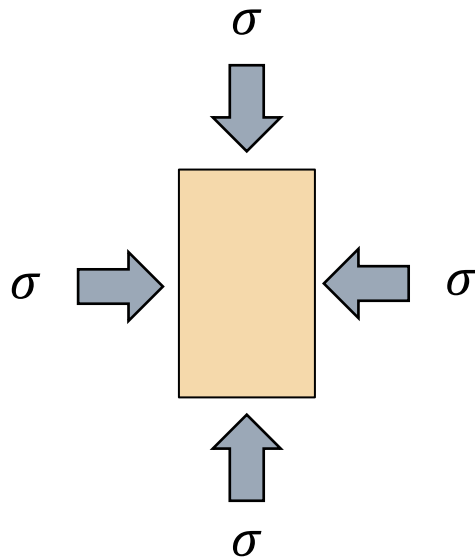
Ring shear test

# Isotropic drained compression

**Principle** → Compression in all direction with the same magnitude

**Use** → Evaluation of the volumetric soil behaviour

**Test** → First stage of a triaxial test: Consolidation phase



**Isotropic stress state**

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma = p$$

$$\sigma'_1 = \sigma'_2 = \sigma'_3 = \sigma' = p'$$

$$q = 0$$

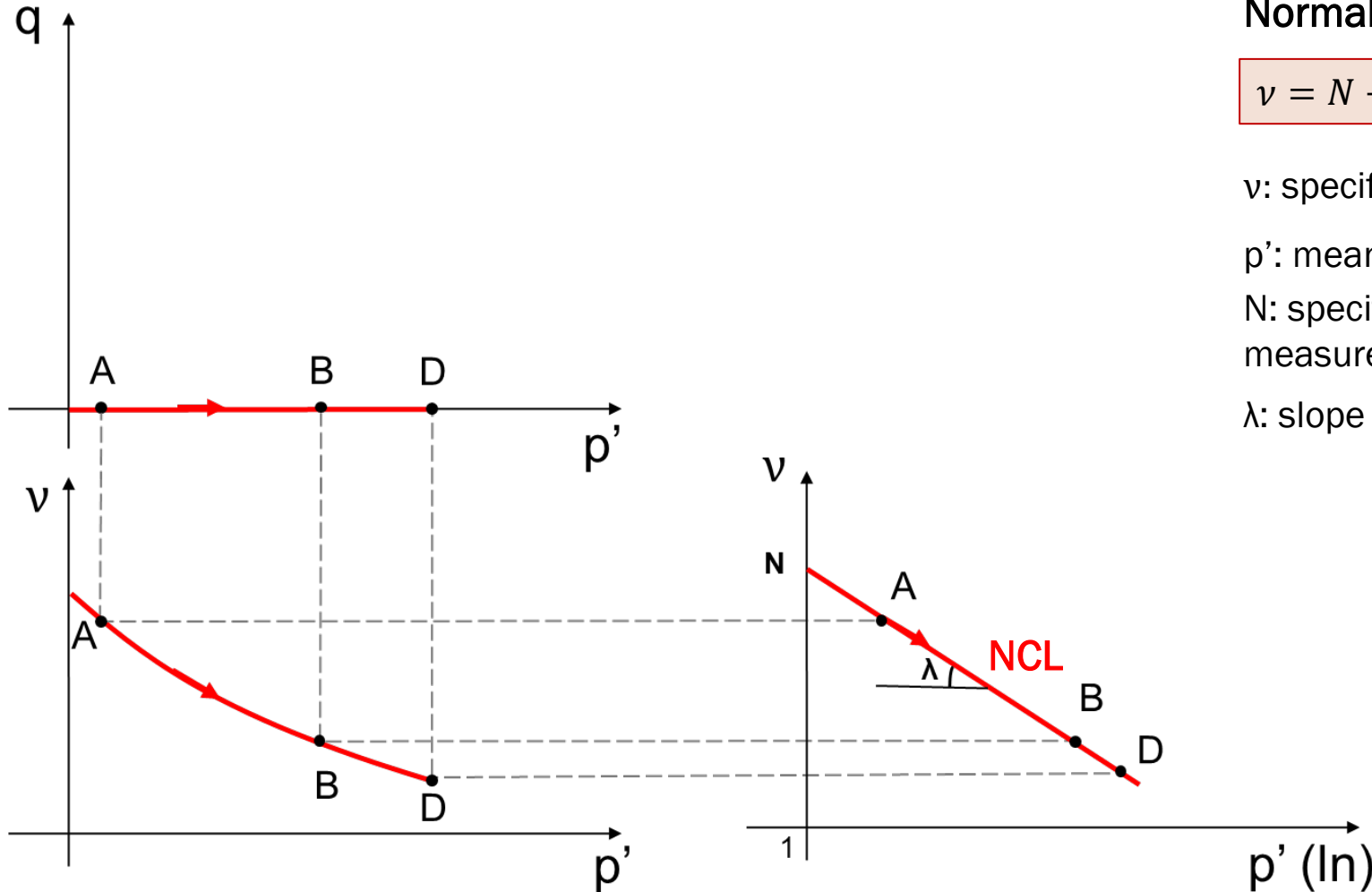
**Drained condition**

$$\Delta p_w = 0 \quad \Rightarrow \quad \Delta p = \Delta p'$$

$$\text{If } p_{w,0} = 0 \quad \Rightarrow \quad p = p'$$

# Isotropic drained compression

## Stress path and stress-specific volume relation



### Normal Compression Line (NCL)

$$v = N - \lambda \ln p'$$

$v$ : specific volume ( $v = 1 + e$ )

$p'$ : mean effective stress

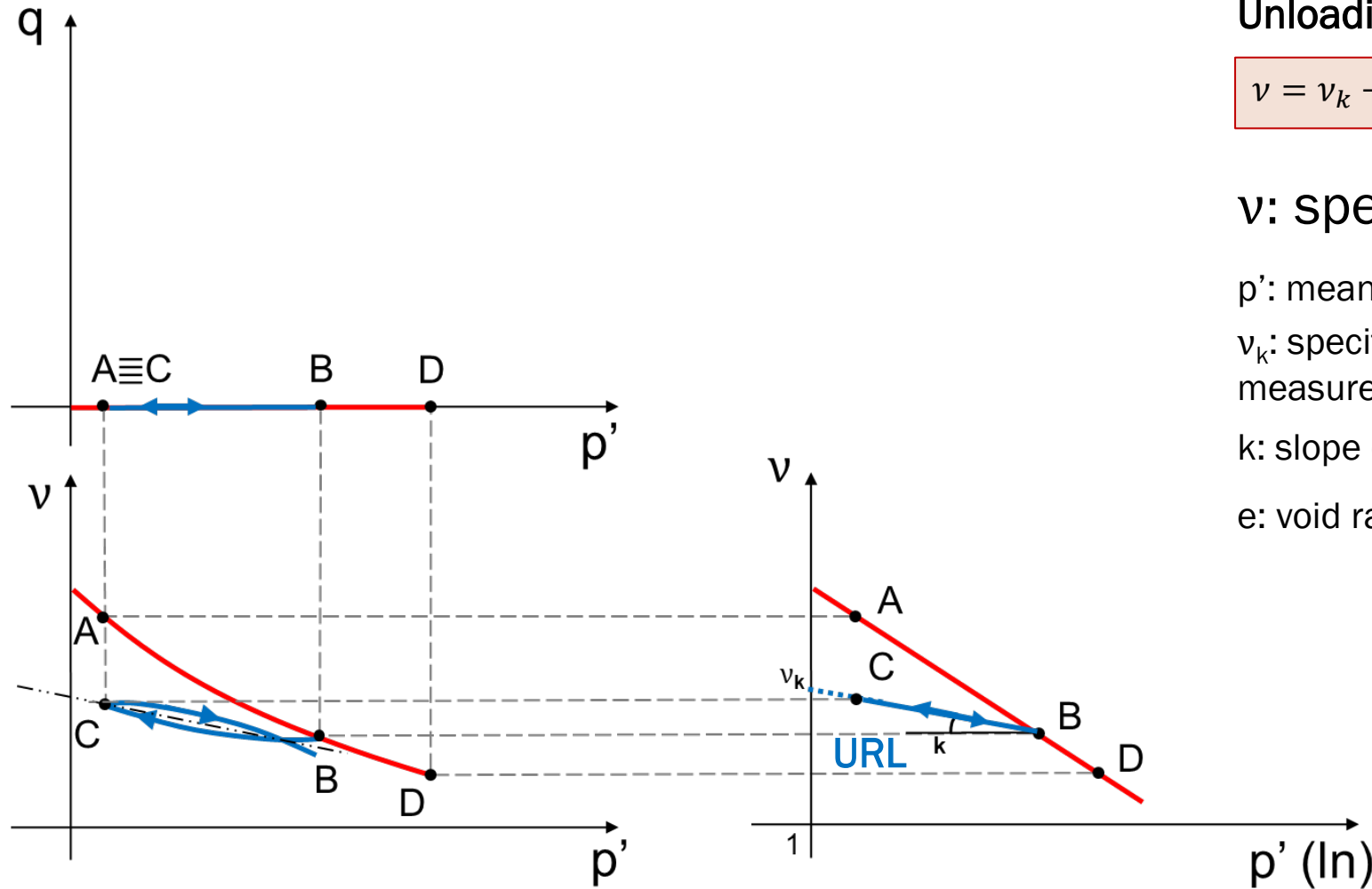
$N$ : specific volume at  $p' = 1$  in the adopted measurement system

$\lambda$ : slope of the NCL (plastic compressibility)

The **NCL** is unique for a given soil

# Isotropic drained compression

## Stress path and stress-specific volume relation



### Unloading Reloading Line (URL)

$$v = v_k - k \ln p'$$

$v$ : specific volume ( $v = 1 + e$ )

$p'$ : mean effective stress

$v_k$ : specific volume at  $p'=1$  in the adopted measurement system

$k$ : slope of the URL (elastic compressibility)

$e$ : void ratio

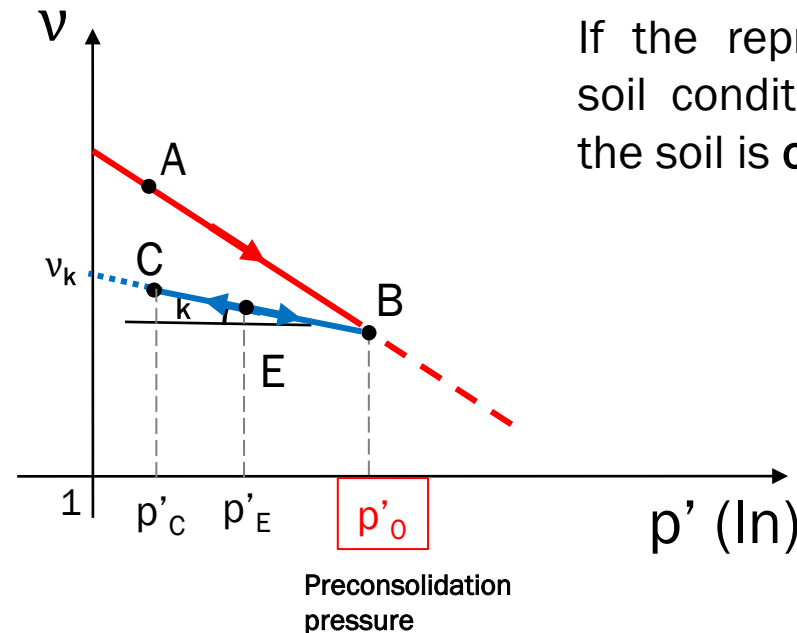
# Isotropic drained compression

## Preconsolidation pressure $p'_0$ and the Overconsolidation ratio OCR

- The **preconsolidation pressure** ( $p'_0$ ):  
Maximum mean effective stress to which the soil has been subjected during the past
- The **overconsolidation ratio (OCR)**  
Ratio between the preconsolidation pressure and the actual mean effective stress

$$OCR = \frac{p'_0}{p'}$$

$p'_0$ : preconsolidation pressure  
 $p'$ : actual mean effective stress



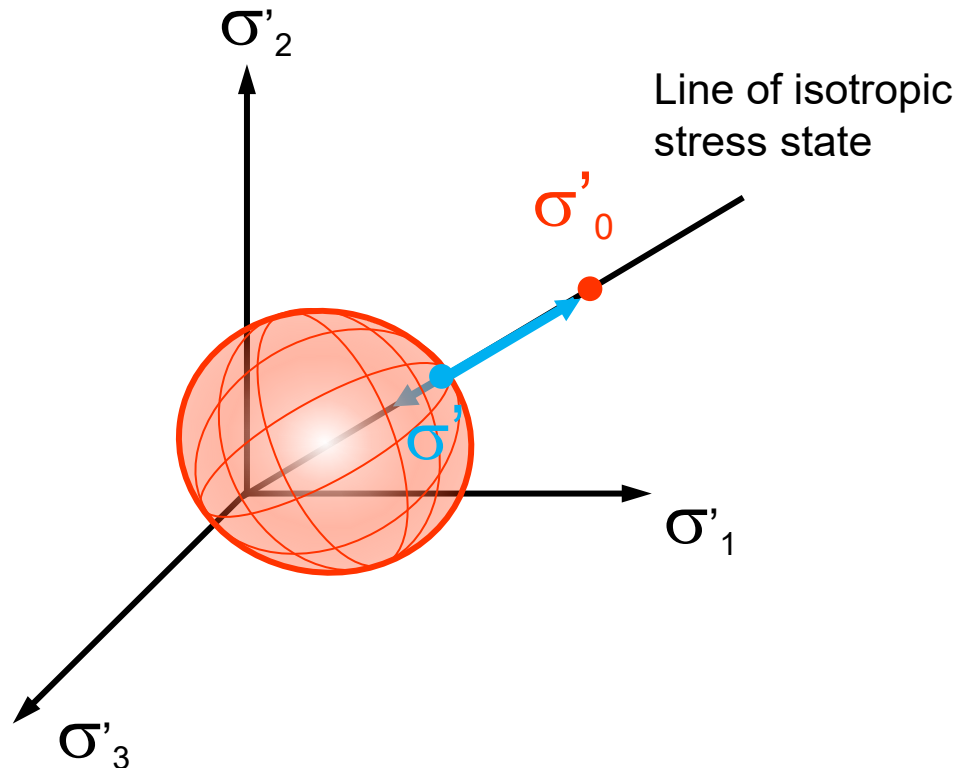
If the representative point of the soil condition belongs to the **URL**, the soil is **overconsolidated** ( $OCR > 1$ )

Example:

$$OCR_E = \frac{p'_0}{p'_E} > 1 \quad OCR_C = \frac{p'_0}{p'_C} > 1 \quad \text{with} \quad OCR_C > OCR_E$$

# Isotropic drained compression

Preconsolidation pressure  $p'_0$  and the Overconsolidation ratio OCR



- Isotropic drained compression:

The sample is first subjected to an isotropic compression  $\sigma'_0$  followed by unloading to  $\sigma$  (OCR > 1)

→ Increase of the elastic domain

→ Density - dependent behaviour

- Introduction
- Experimental evidences Conventional Triaxial Compression tests
  - Isotropic drained compression
  - **Shearing** phase on a **NC** soil in **drained and undrained** conditions
  - Shearing phase on a OC soil in drained and undrained conditions
- Critical state and shear strength
- Examples

# Shearing a **NC soil in drained** compression

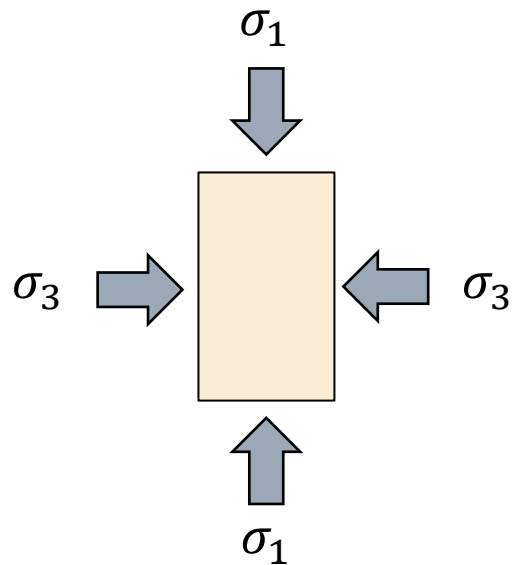
**Principle** → Increase of the axial stress and imposed drained conditions

**Use** → Evaluation of the soil shear strength

**Test** → Second stage of a triaxial test with drainage valves open

On the **NCL**:

$$OCR = \frac{p'_0}{p'} = 1$$



Stress state variables

$$p = \frac{\sigma_1 + 2\sigma_3}{3}$$

$$p' = \frac{\sigma'_1 + 2\sigma'_3}{3}$$

$$q = \sigma_1 - \sigma_3$$

Drained condition

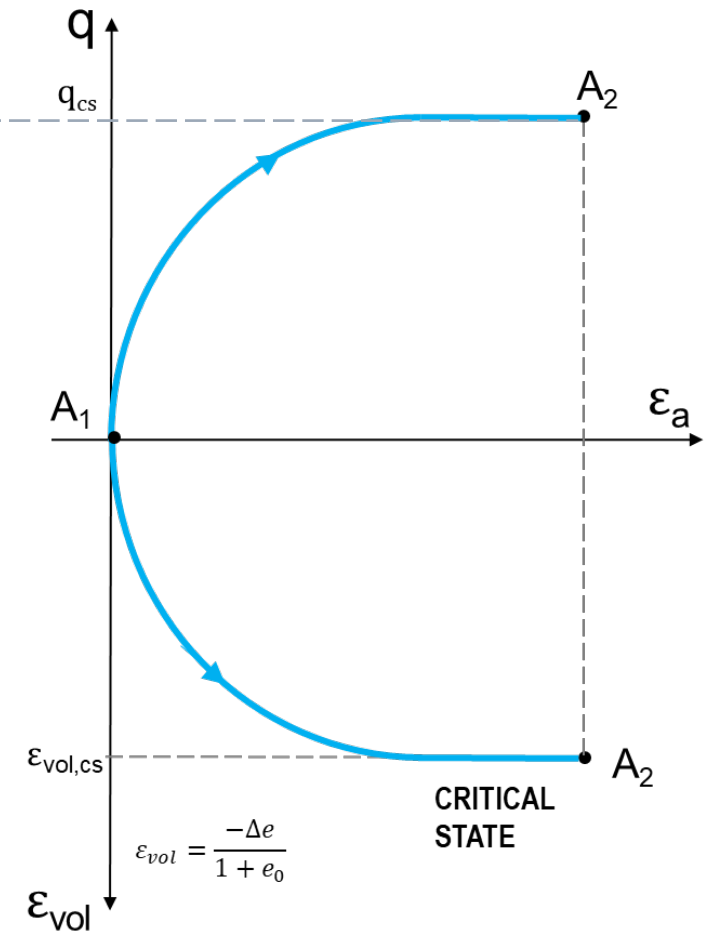
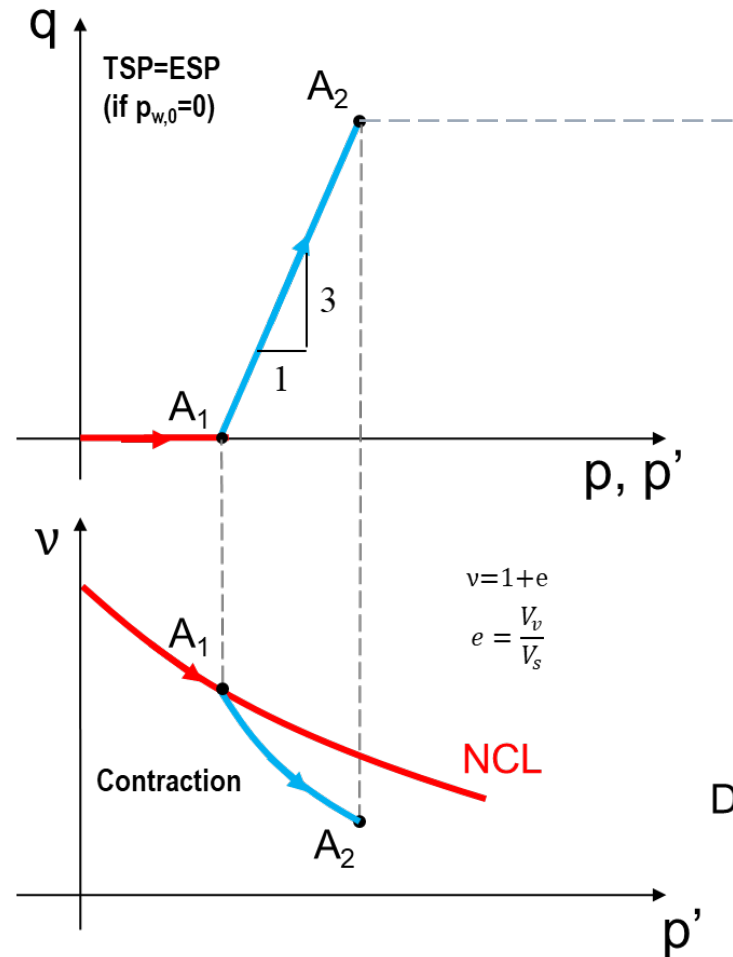
$$\Delta p_w = 0 \quad \Rightarrow \quad \Delta p = \Delta p'$$

$$\text{If } p_{w,0} = 0 \quad \Rightarrow \quad p = p'$$

# Shearing a **NC soil in drained** compression

## Stress path and stress-strain response

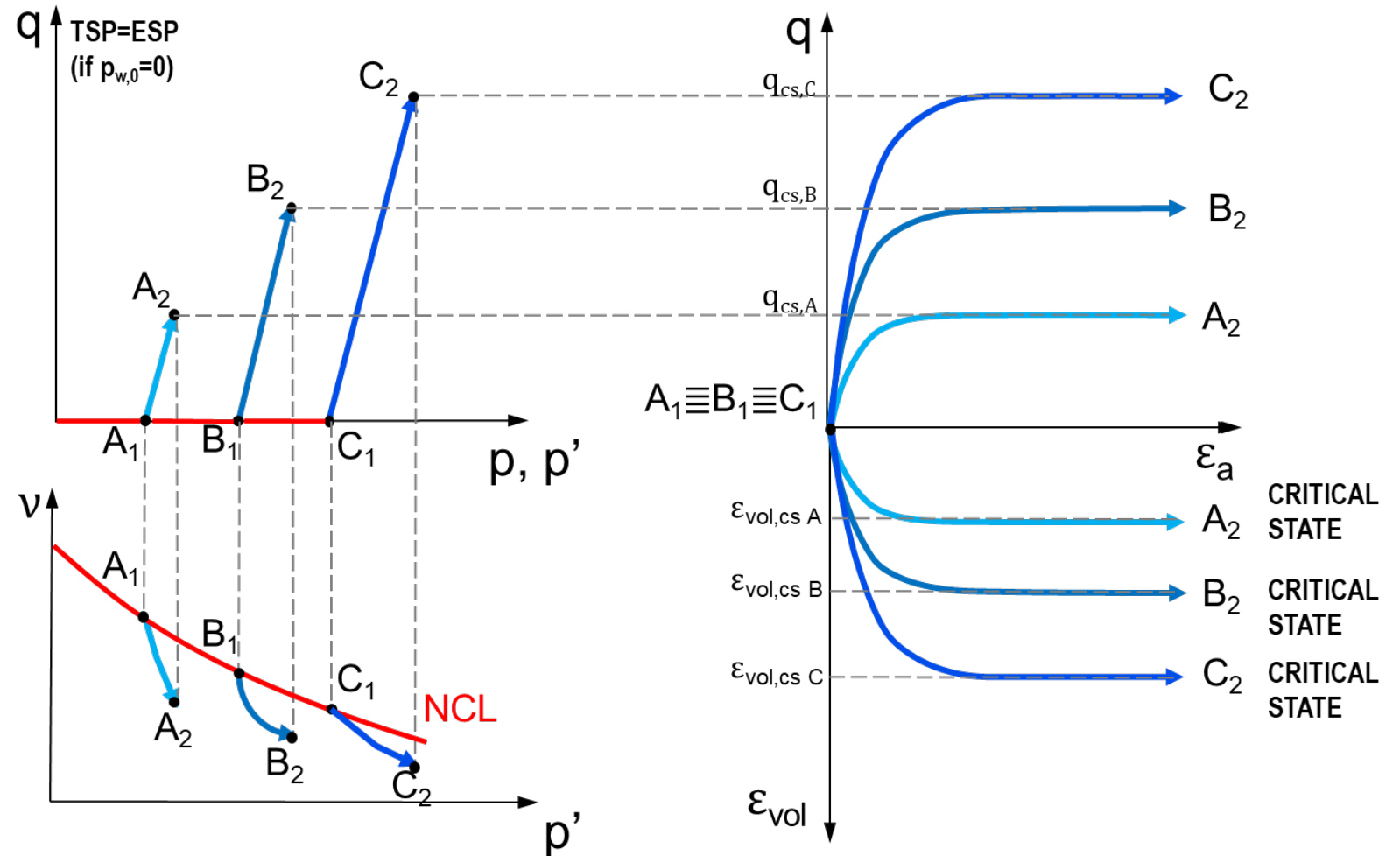
A single sample (A)



# Shearing a **NC soil in drained** compression

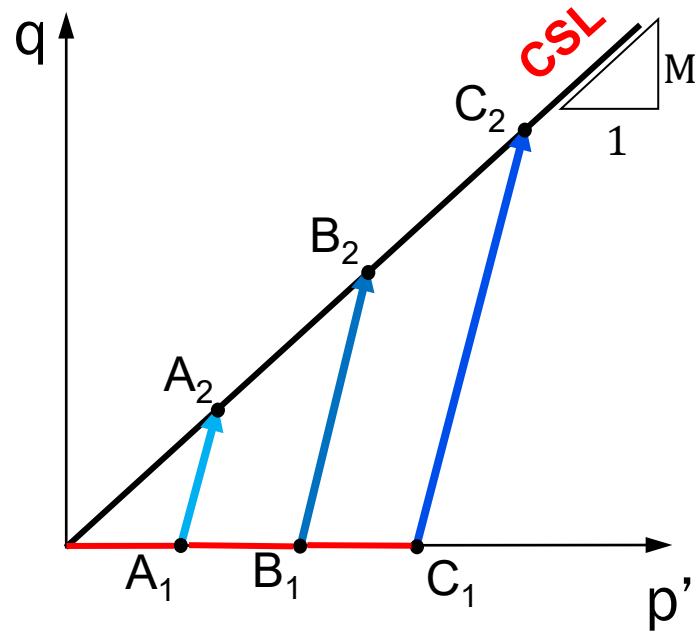
## Stress path and stress-strain response

Three samples (A,B,C)  
consolidated at three  
different confinement



# Shearing a **NC soil in drained** compression

## Critical State Line



In the  $q - p'$  plane

All points representative of the soil condition at the **critical state** ( $A_2, B_2, C_2$ ) in the  $q-p'$  plane are well interpretable by a **straight line** passing through the origin

$$q = Mp'$$

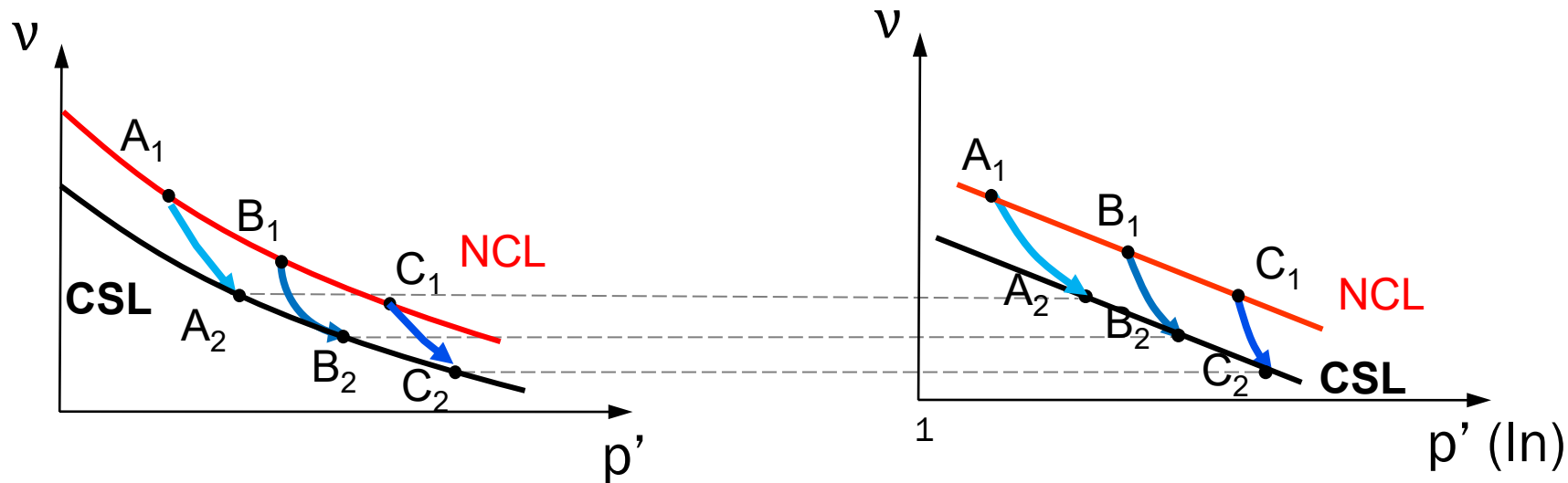
$p'$ : Mean effective stress

$q$ : Deviatoric stress

$M$ : Slope of the CSL in the  $q-p'$  plane

# Shearing a **NC soil in drained** compression

Critical State Line (CSL) in the  $v - p'$  plane



All points representative of the soil condition at the **critical state** ( $A_2, B_2, C_2$ ) are well interpretable by a **straight line in the  $v - p'(\ln)$  plane** :

$$v = \Gamma - \lambda \ln p'$$

- $v$  : specific volume ( $v=1+e$ )
- $p'$  : mean effective stress
- $\Gamma$  : specific volume at  $p'=1$  in the adopted measurement system
- $\lambda$  : slope of the NCL and of the CSL (plastic compressibility)

# Shearing a **NC soil in undrained** compression

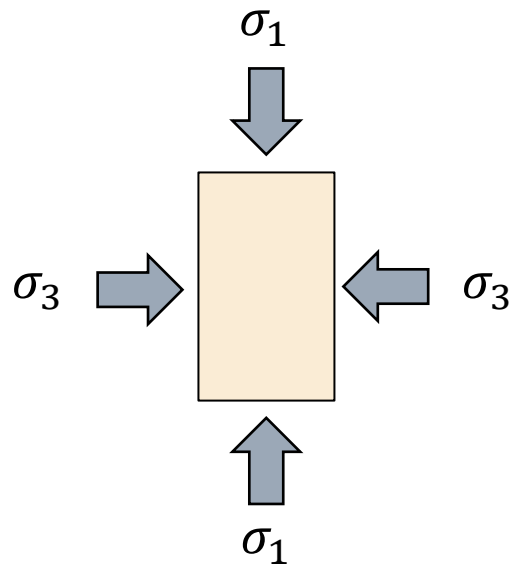
**Principle** → Increase of the axial stress and imposed undrained conditions

**Use** → Evaluation of the soil shear strength

**Test** → Second stage of a triaxial test with drainage valves closed

On the **NCL**:

$$OCR = \frac{p'_0}{p'} = 1$$



Stress state variables

$$p = \frac{\sigma_1 + 2\sigma_3}{3}$$

$$p' = \frac{\sigma'_1 + 2\sigma'_3}{3}$$

$$q = \sigma_1 - \sigma_3$$

Undrained condition


$$\Delta p_w \neq 0 \quad \Rightarrow \quad \Delta p \neq \Delta p'$$

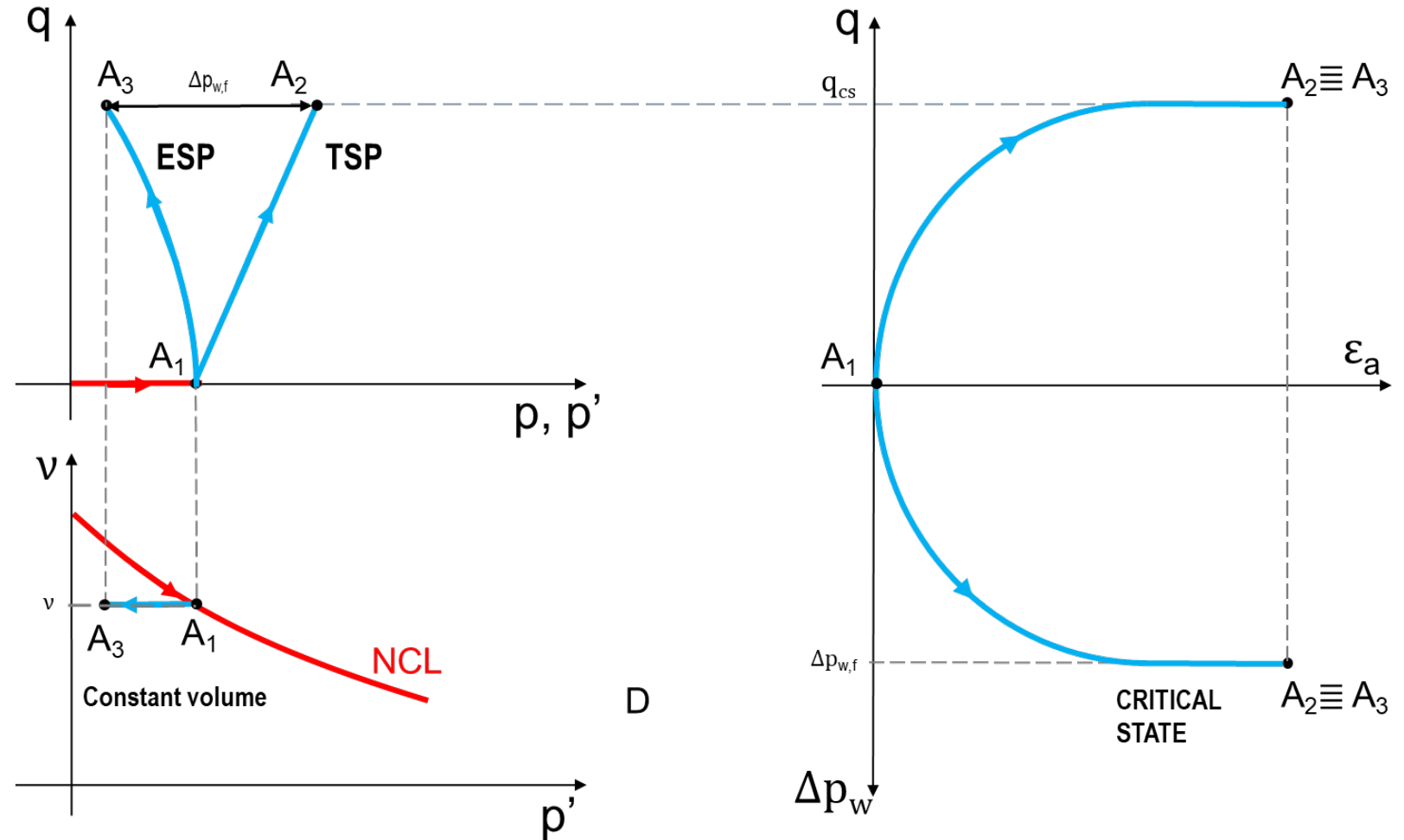
$$\Delta V = 0 \quad \Rightarrow \quad \varepsilon_v = 0$$

# Shearing a NC soil in **undrained** compression

## Stress path and stress-strain response

A single sample (A)

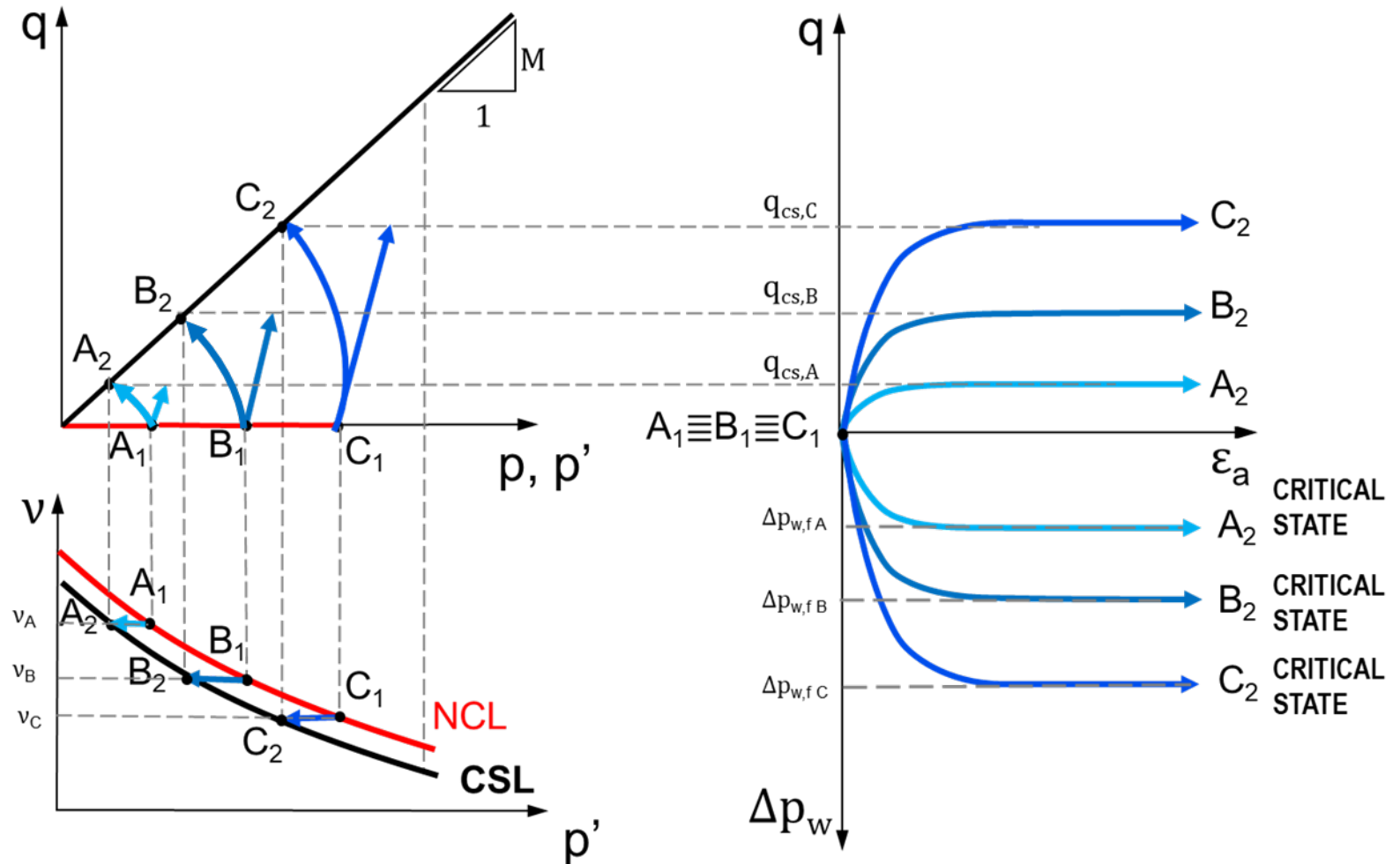
  $A_1$  is the starting point of both the effective and total stress paths only if  $p_{w,0}=0$



# Shearing a NC soil in **undrained** compression

## Stress path and stress-strain response

Three samples (A,B,C) consolidated at three different confinement



# Shearing a NC soil

## Critical State Lines in drained and undrained conditions

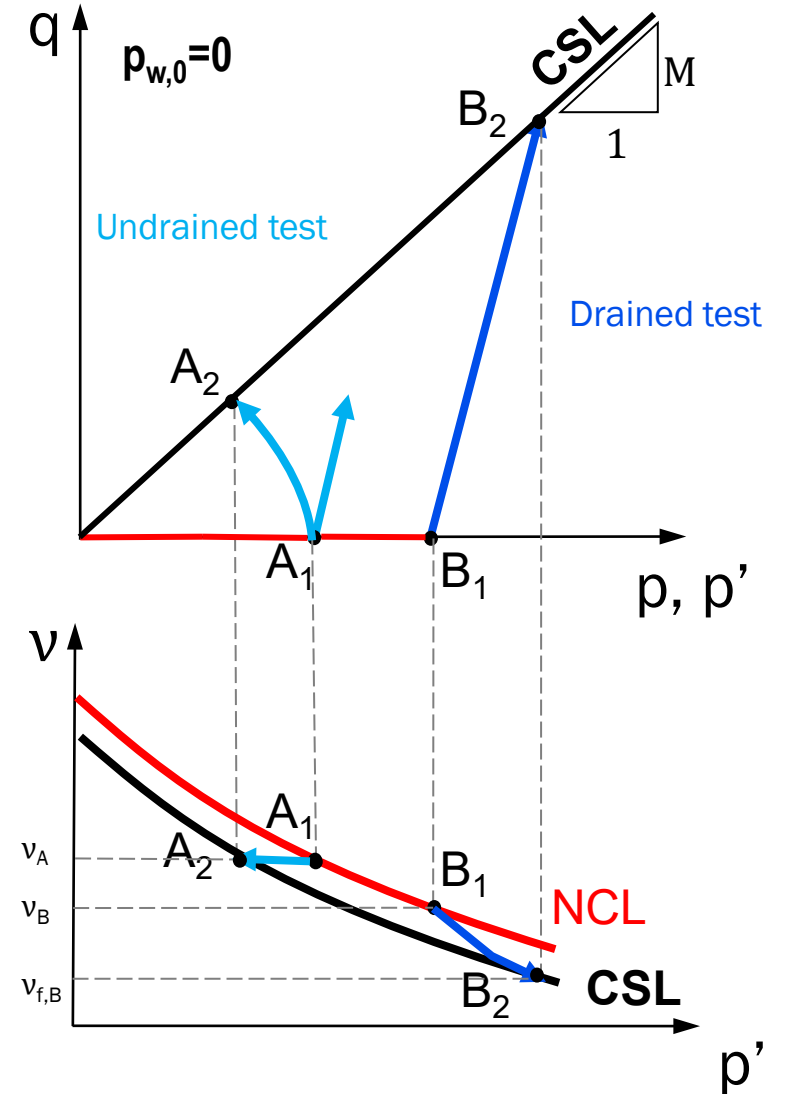
The **Critical State Lines** do not depend on the test conditions (drained or undrained)

$$q = Mp'$$

$$v = \Gamma - \lambda \ln p'$$

➤ If the soil is a NC clay (or a loose sand), the CSL is reached with a contracting behavior:

- If  $V$  can change (**drain. cond.**)
  - ➔  $V$  decrease during shear ( $B_1$ - $B_2$ )
- If  $V$  cannot change (**undrain. cond.**)
  - ➔  $\Delta p_w > 0$  during shear ( $A_1$ - $A_2$ )



# Critical State Line in $q$ - $p'$ - $v$ plane

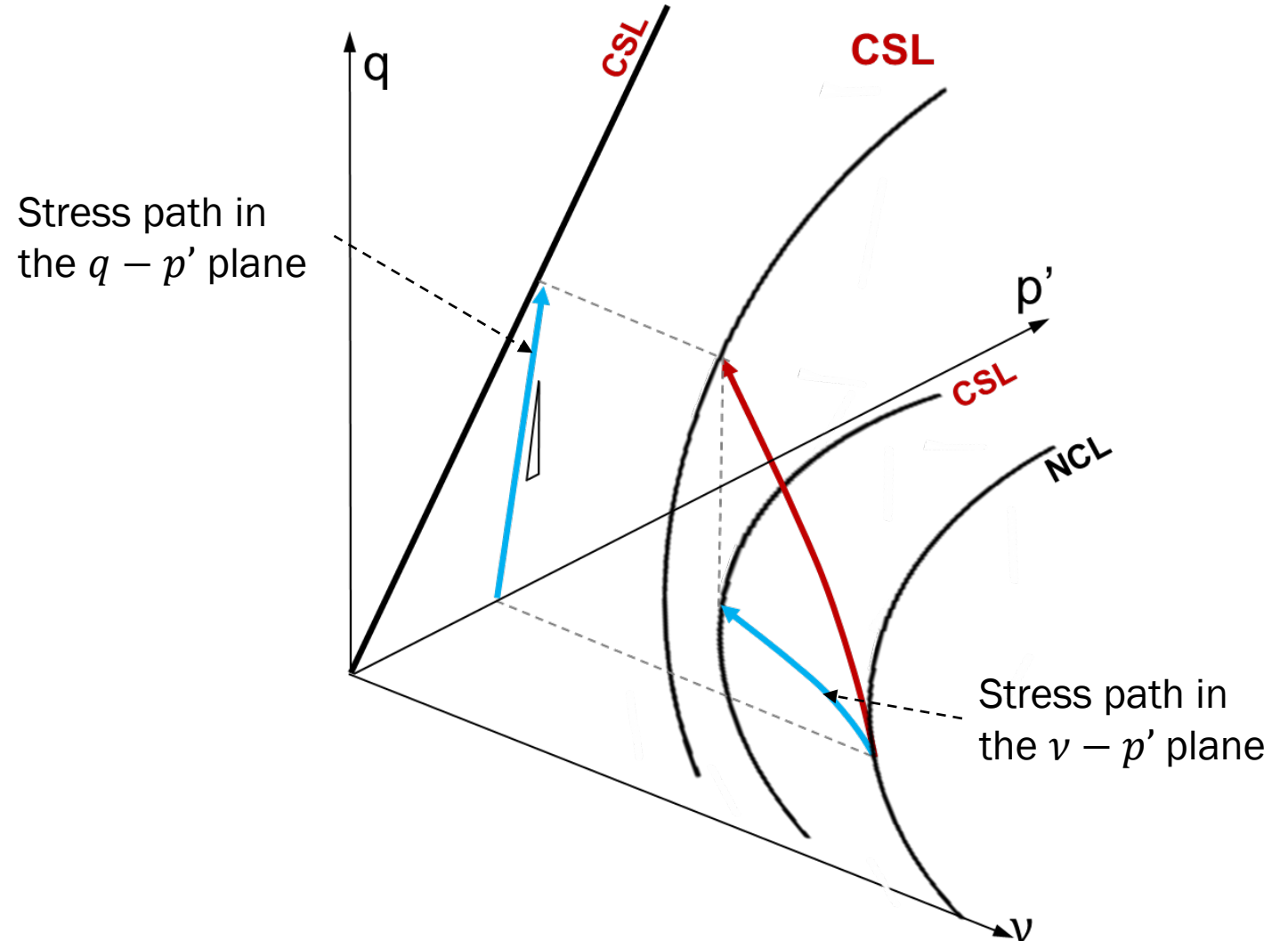
Combination of planes: Drained path

- $q - p'$  plane

$$q = Mp'$$

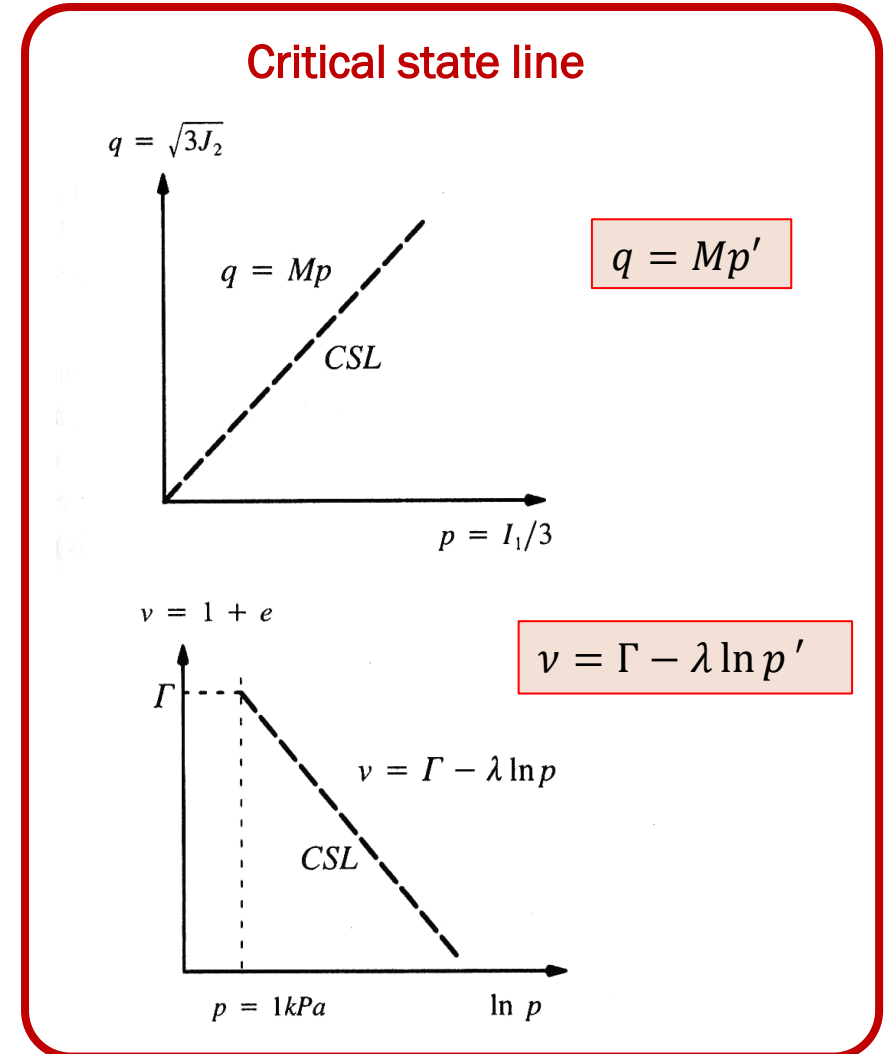
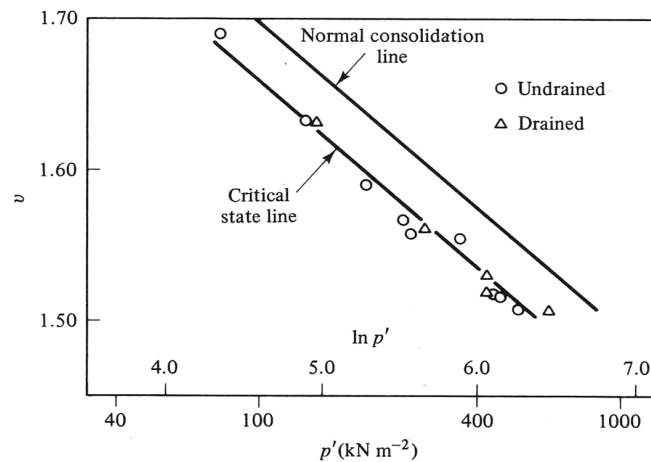
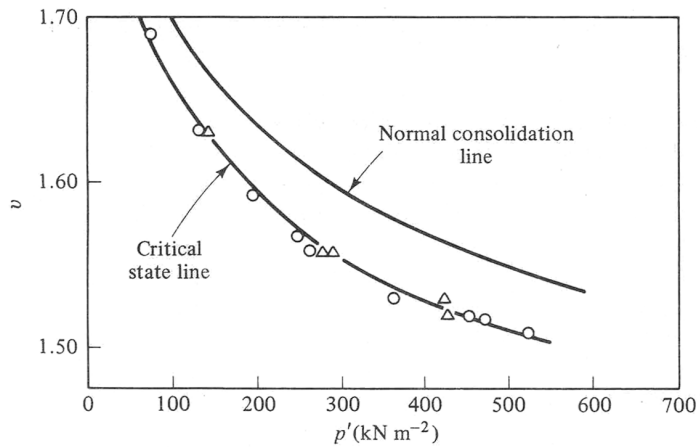
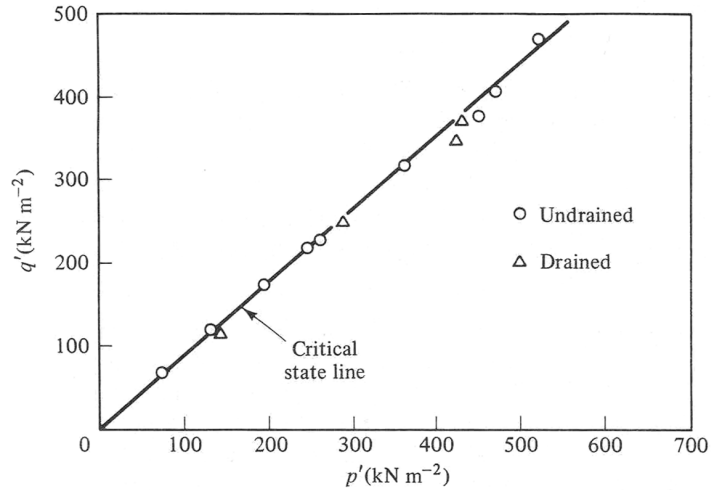
- $q - v$  plane

$$v = \Gamma - \lambda \ln p'$$



# Critical state Line

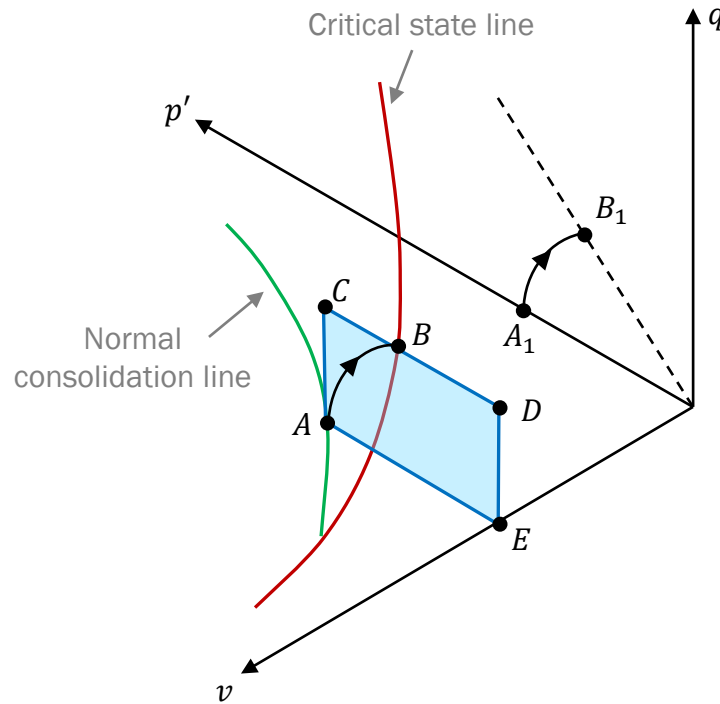
## Combination of drained and undrained tests



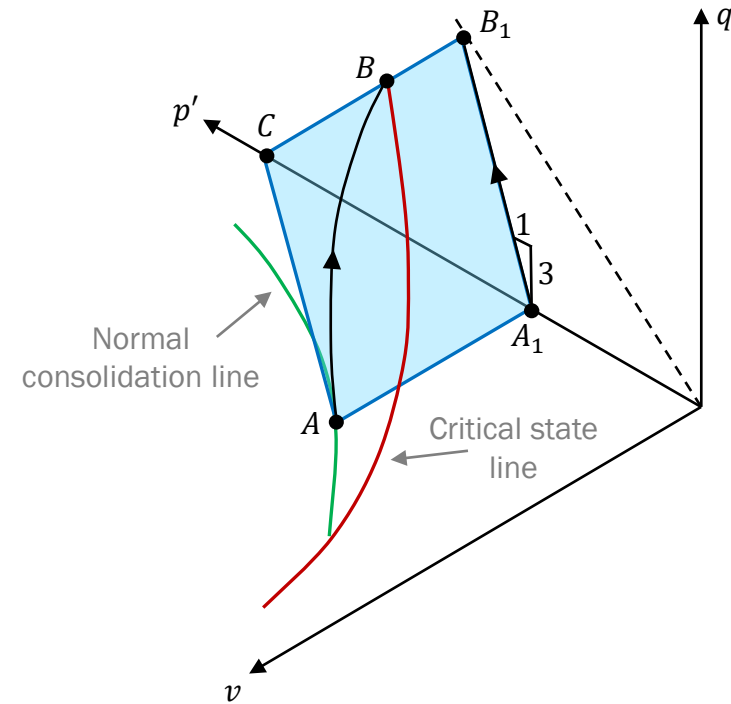
# CSL: Drained & undrained path in $q - p' - v$

**Drained & Undrained:** Triaxial CTC on normally consolidated samples until failure

**Undrained**



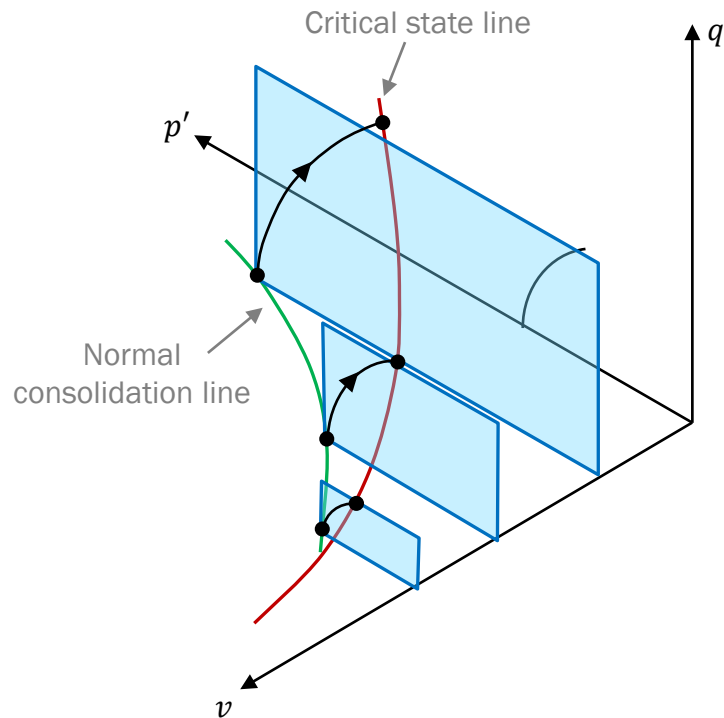
**Drained**



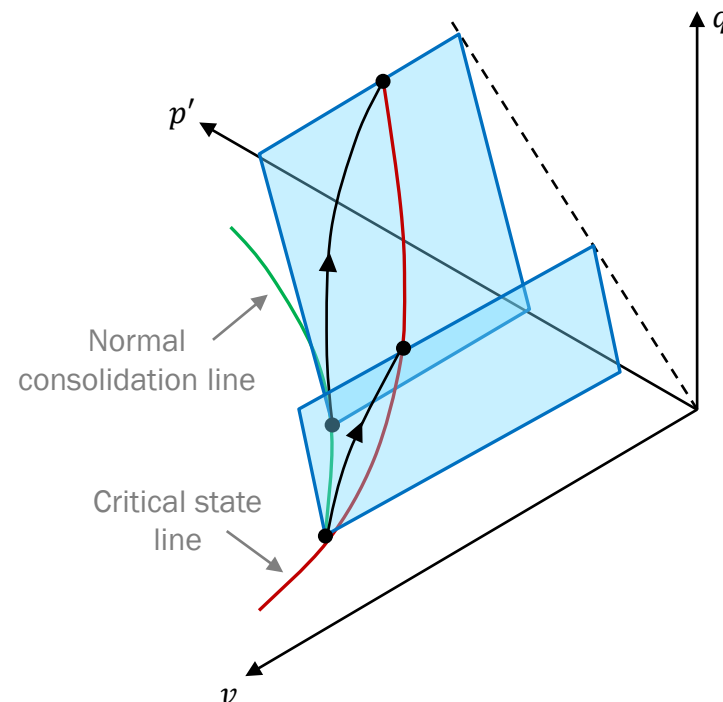
# CSL: Families of drained & undrained tests in $q - p' - v$

**Drained & Undrained:** Triaxial CTC on normally consolidated samples until failure

**Undrained**



**Drained**



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  - **Shearing** phase on a **OC** soil in **drained and undrained** conditions
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# Shearing a **OC soil in drained** compression

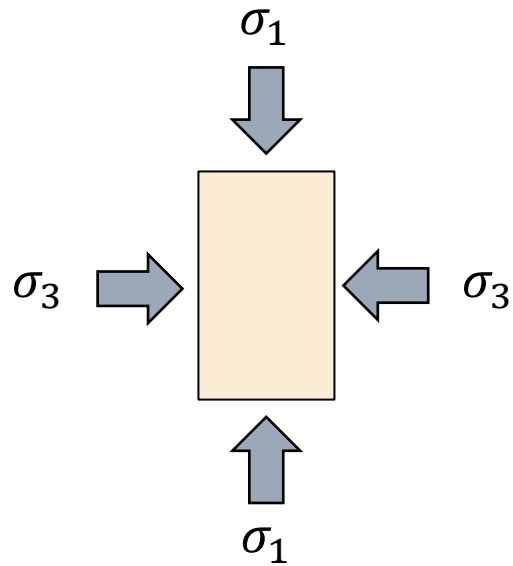
**Principle** → Increase of the axial stress and imposed drained conditions

**Use** → Evaluation of the soil shear strength

**Test** → Second stage of a triaxial test with drainage valves open

On the **URL**:

$$OCR = \frac{p'_0}{p'} > 1$$



Stress state variables

$$p = \frac{\sigma_1 + 2\sigma_3}{3}$$

$$p' = \frac{\sigma'_1 + 2\sigma'_3}{3}$$

$$q = \sigma_1 - \sigma_3$$

Drained condition

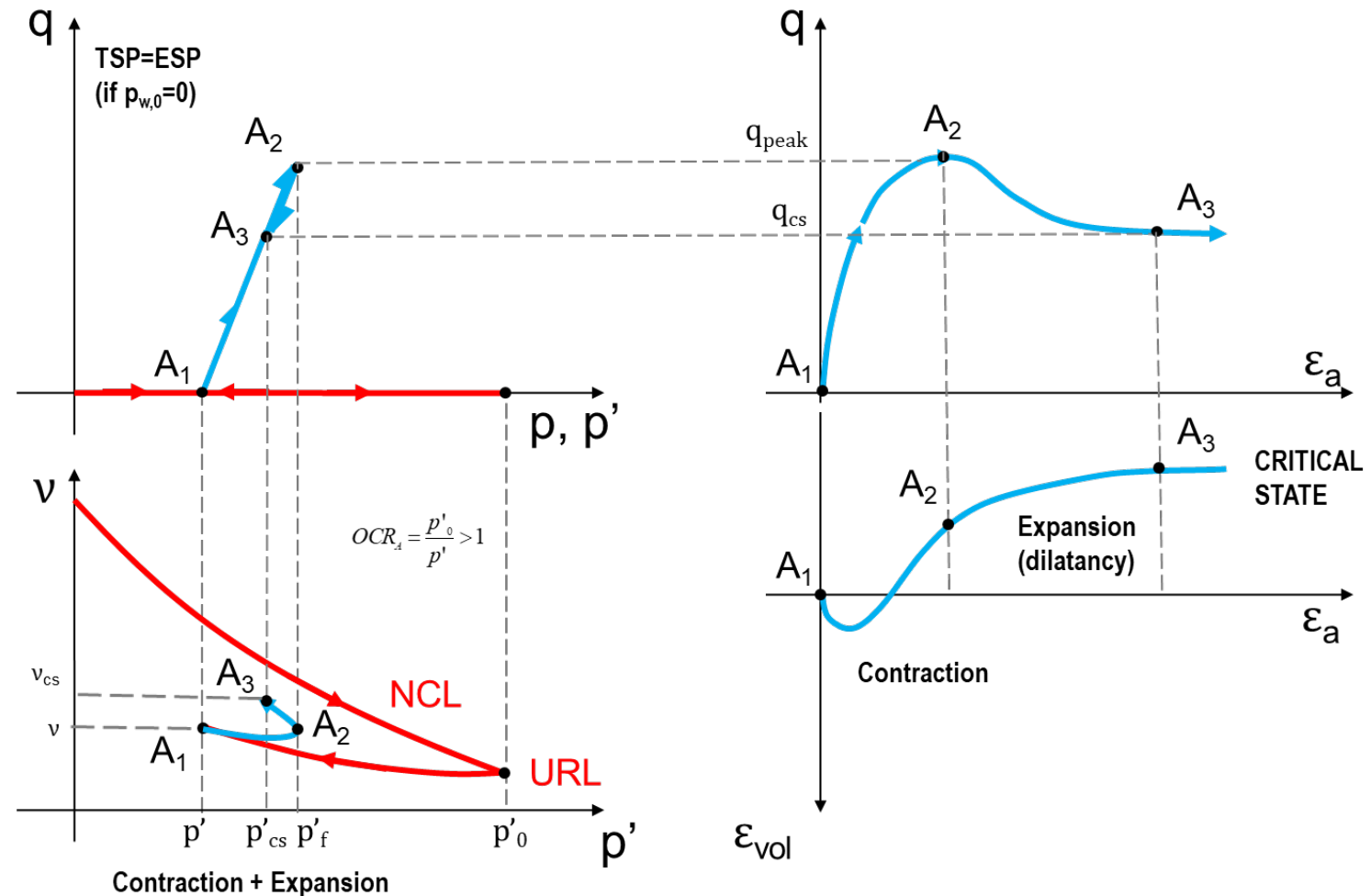
$$\Delta p_w = 0 \quad \Rightarrow \quad \Delta p = \Delta p'$$

$$\text{If } p_{w,0} = 0 \quad \Rightarrow \quad p = p'$$

# Shearing an OC soil in **drained** compression

## Stress path and stress-strain response

A single sample (A)

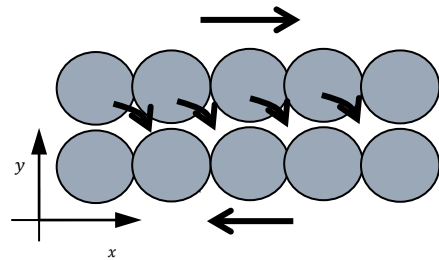


# Shearing an OC soil in **drained** compression

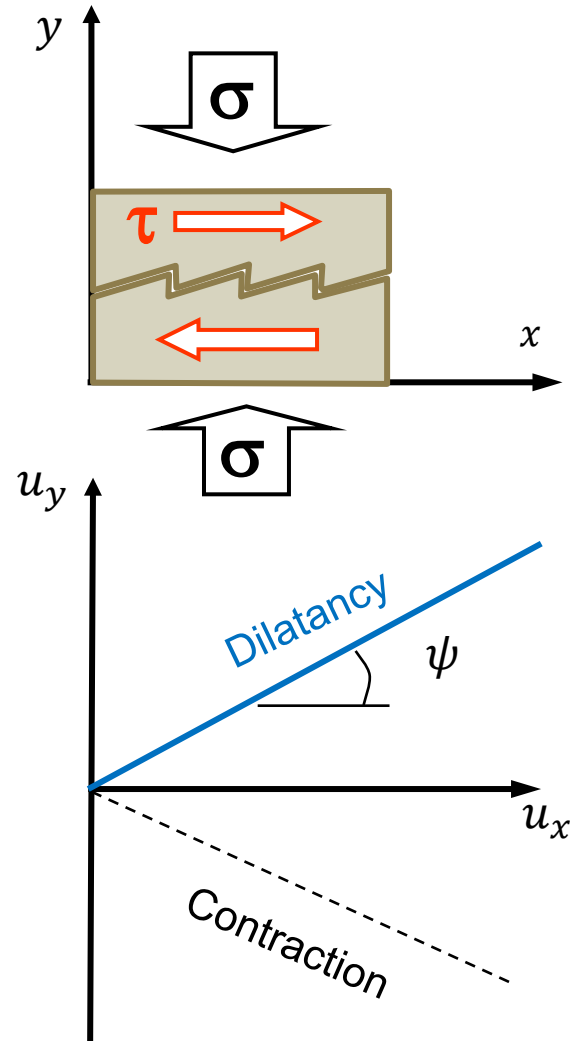
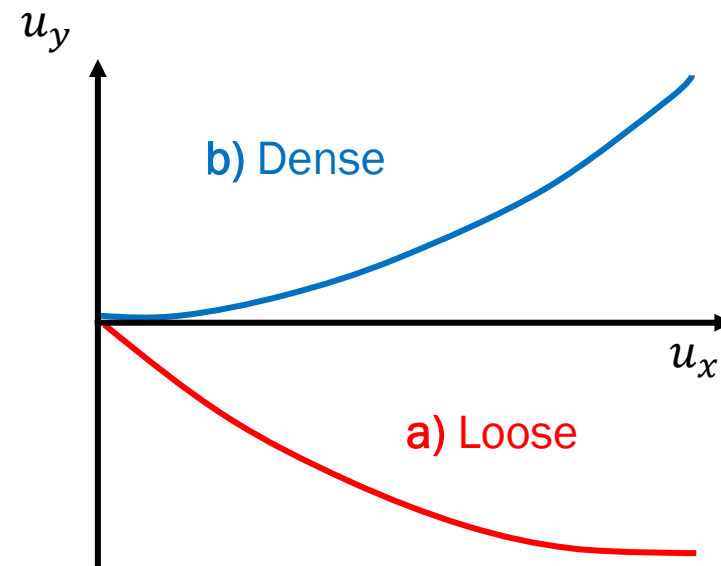
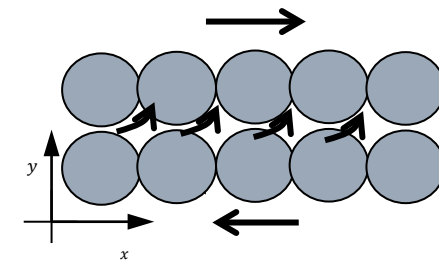
## Dilatancy and contractive behaviour

➤ The **density** defines the **contractive** or **dilating** behaviour of the soil

a) Shearing of **loosely packed** layers of circular discs

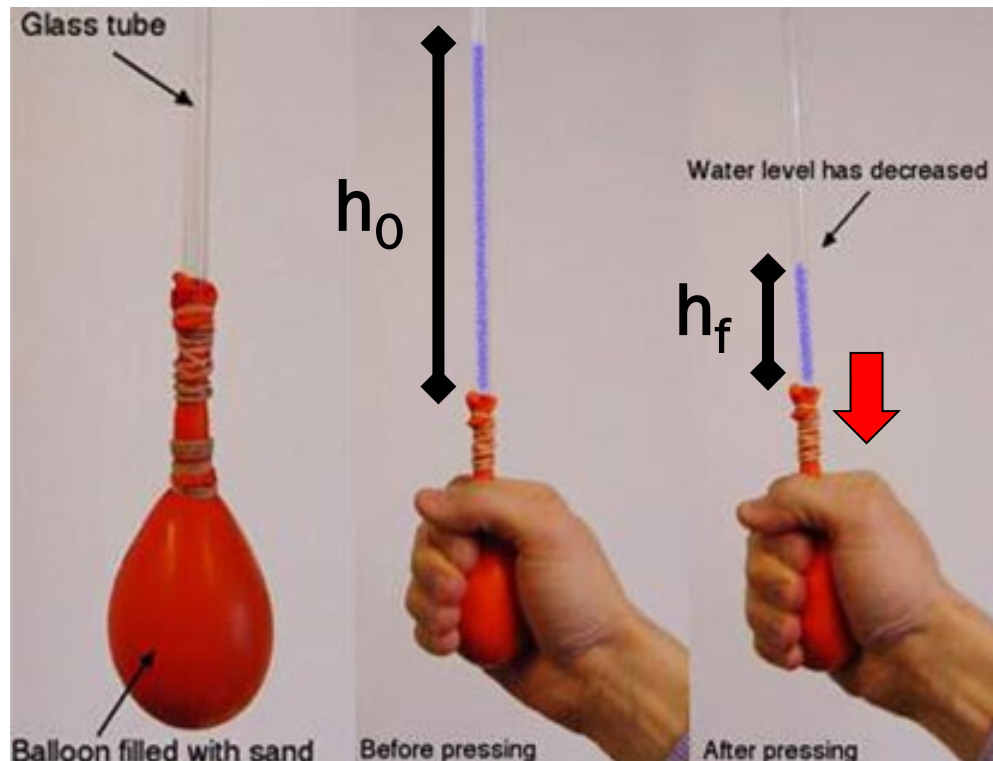


b) Shearing of **densely packed** layers of circular discs



# Shearing an OC soil in **drained** compression

Reynolds's experiments (1885) - Dilatancy



- A balloon is filled with dense sand.
- A glass tube is filled with water.

→ Pressing the balloon, water goes down on the tube.

Well compacted sand → tendency to expand

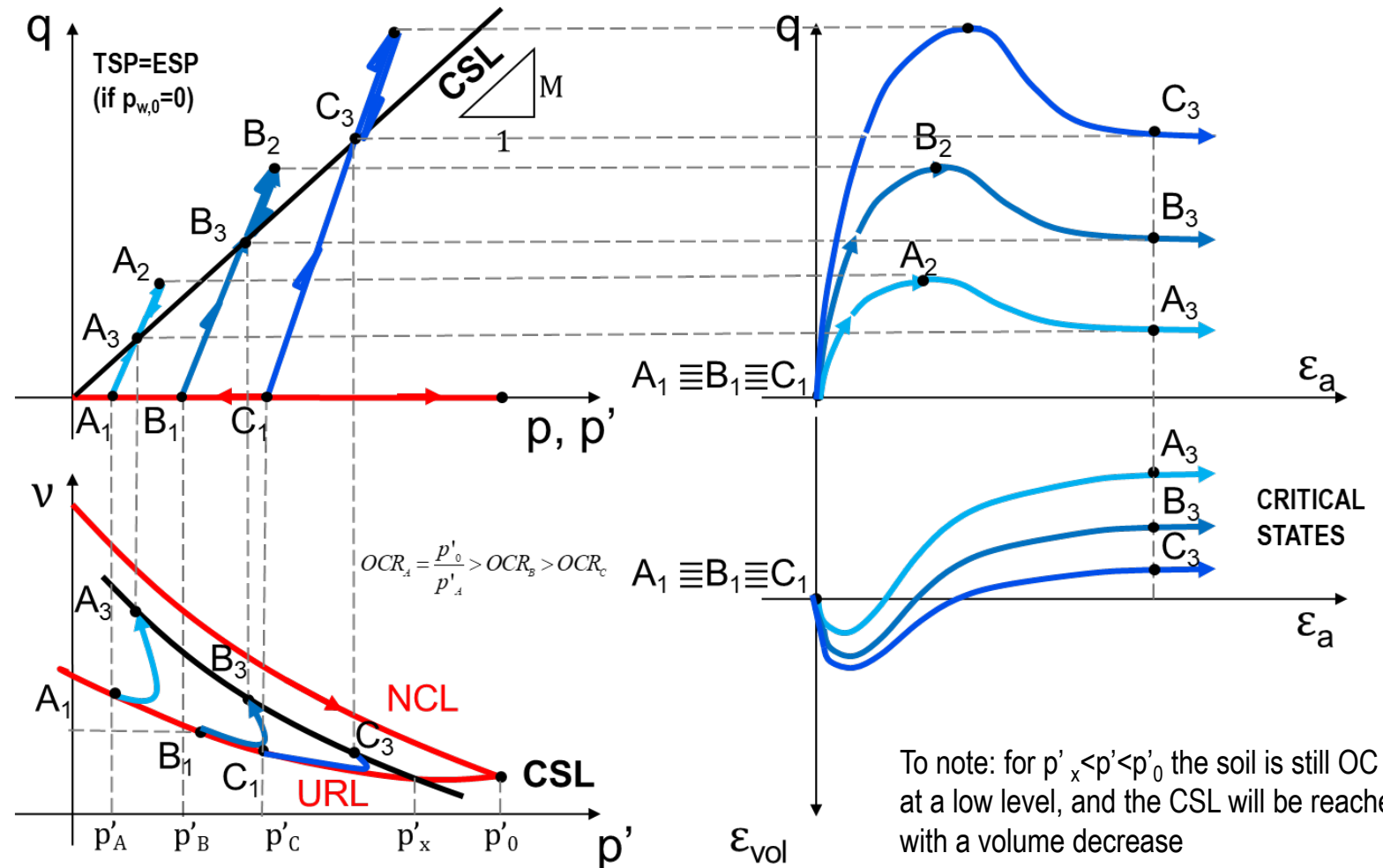
- the space between grains increase
- The upper water to enter within it.

# Shearing an OC soil in drained compression

## Stress path and stress-strain response

Three samples (A,B,C) consolidated at three different confinement

The higher the OCR, the greater the volume increase that must occur for reaching the critical state (higher dilatancy)

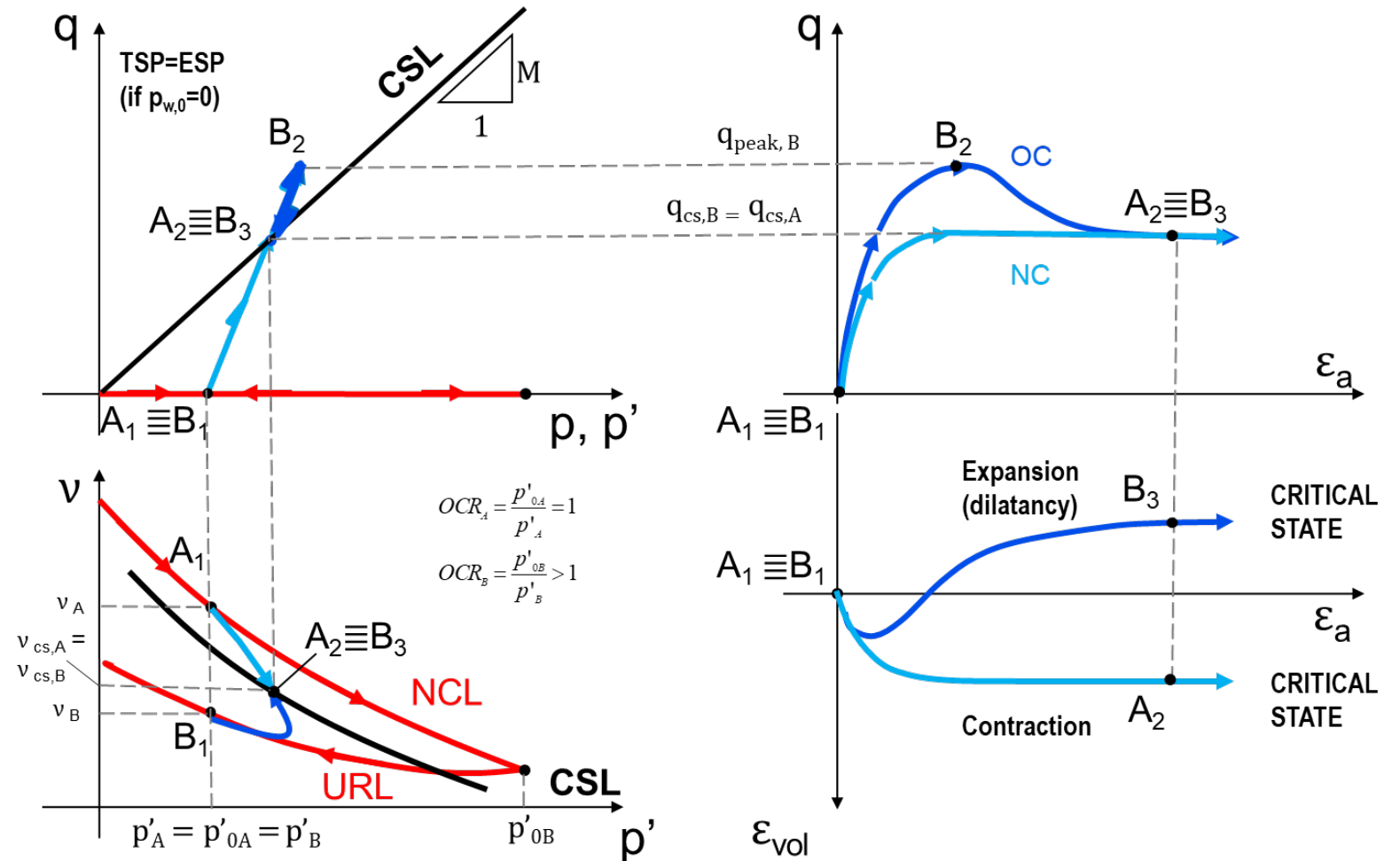


# Shearing in drained compression: NC and OC

## Stress path and stress-strain response

Two different samples:

- A is NC
- B is OC



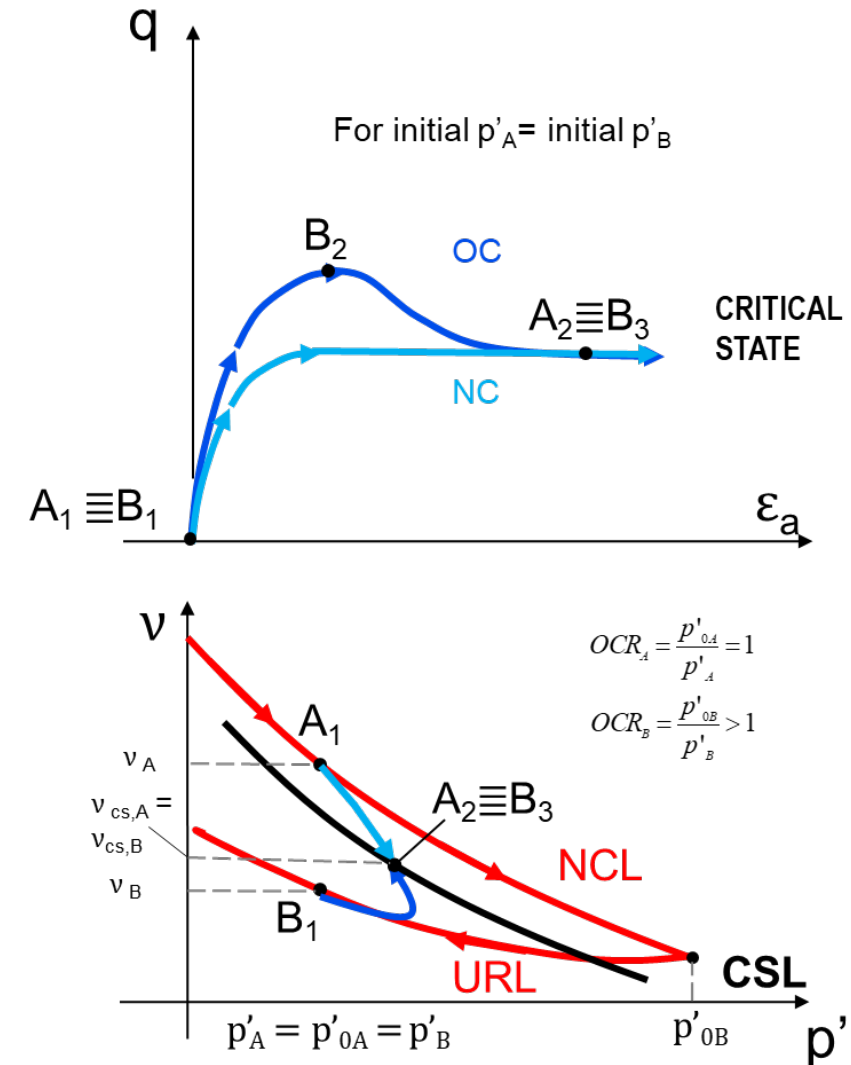
# Shearing in drained compression: NC and OC

## Critical State Line

The **Critical State Lines** do not depend on the test conditions (NC or OC)

➤ In drained conditions the CSL is reached with a contracting behavior:

- If the soil is a **NC** clay (or a loose sand) with a contractive behavior
  - ➔  $\Delta V < 0$  during shear ( $A_1-A_2$ )
- If the soil is a **OC** clay (or a dense sand), with a dilatant behavior
  - ➔  $\Delta V > 0$  during shear ( $B_1-B_3$ )
  - if the soil has a significant OCR (otherwise  $\Delta V < 0$ )



# Shearing a **OC soil in undrained** compression

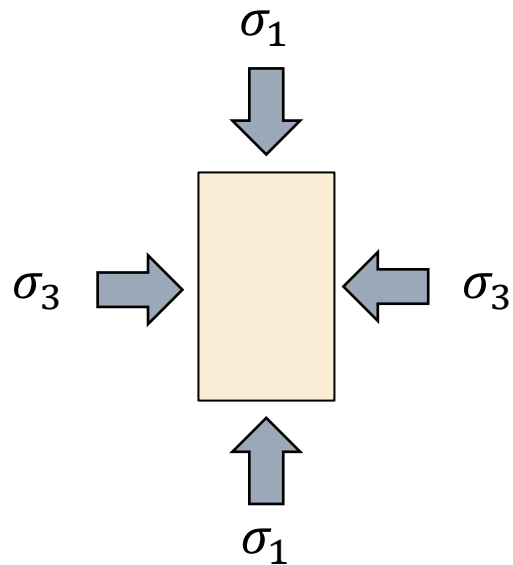
**Principle** → Increase of the axial stress and imposed undrained conditions

**Use** → Evaluation of the soil shear strength

**Test** → Second stage of a triaxial test with drainage valves closed

On the **URL**:

$$OCR = \frac{p'_0}{p'} > 1$$



Stress state variables

$$p = \frac{\sigma_1 + 2\sigma_3}{3}$$

$$p' = \frac{\sigma'_1 + 2\sigma'_3}{3}$$

$$q = \sigma_1 - \sigma_3$$

Undrained condition

$$\Delta p_w \neq 0 \quad \Rightarrow \quad \Delta p \neq \Delta p'$$

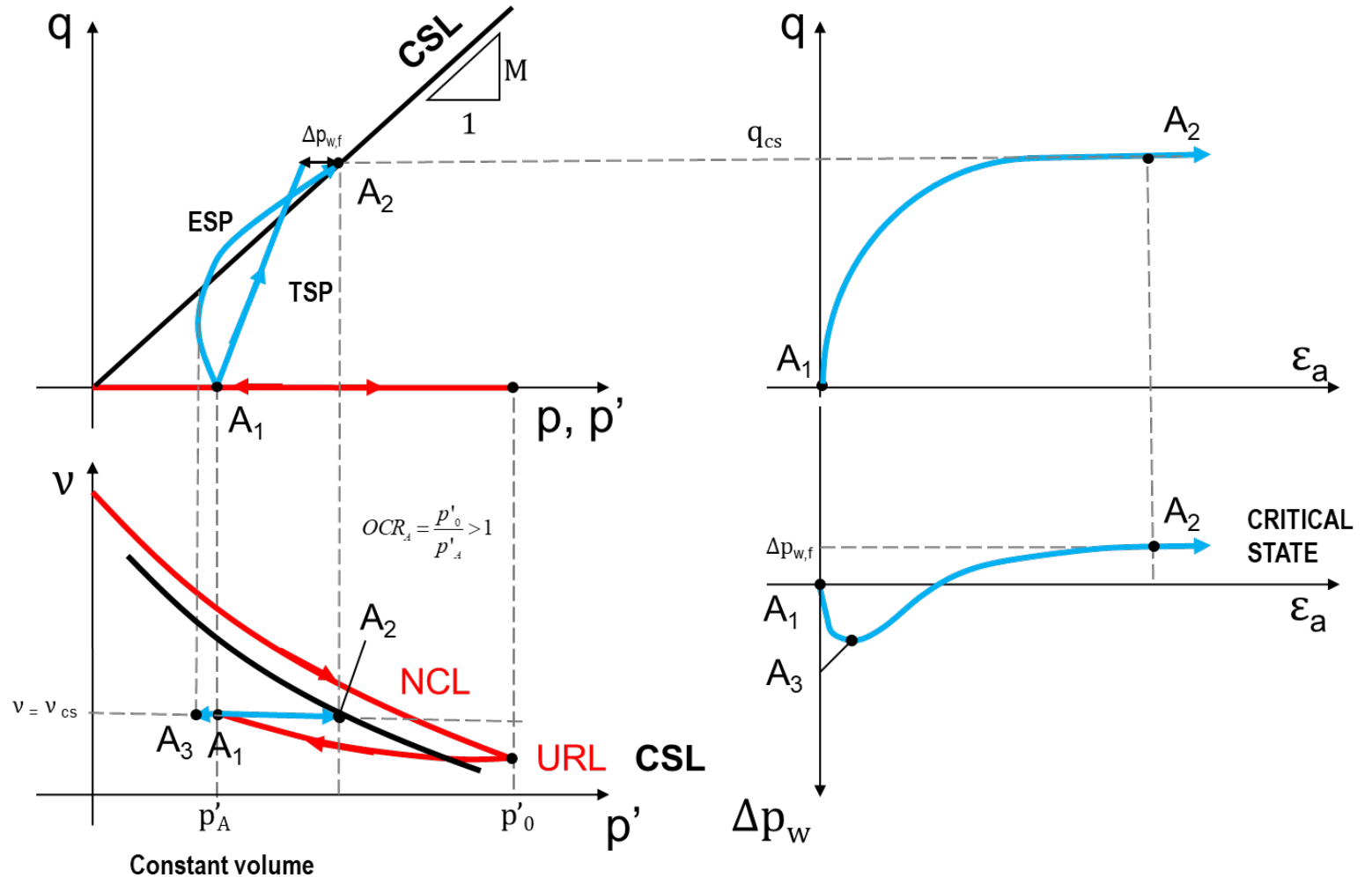
$$\Delta V = 0 \quad \Rightarrow \quad \varepsilon_v = 0$$

# Shearing a OC soil in **undrained** compression

## Stress path and stress-strain response

A single sample (A)

- The **dilating tendency** of a OC soil results in the development of a **negative overpressure** if the volume cannot change (undrained cond.)



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- **Critical state and shear strength**
- Examples

# Critical state and shear strength

## Shear strength definition



Shear strength



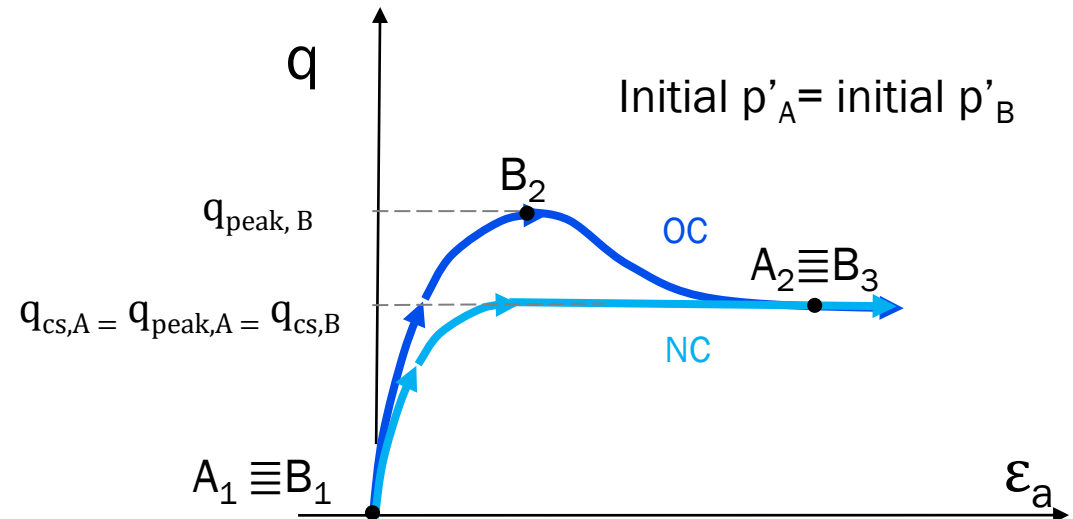
Peak shear strength ( $q_{peak}$ )

Critical (or ultimate) shear strength ( $q_{cs}$ )

→ Only **OC** soil shows **peak** strength

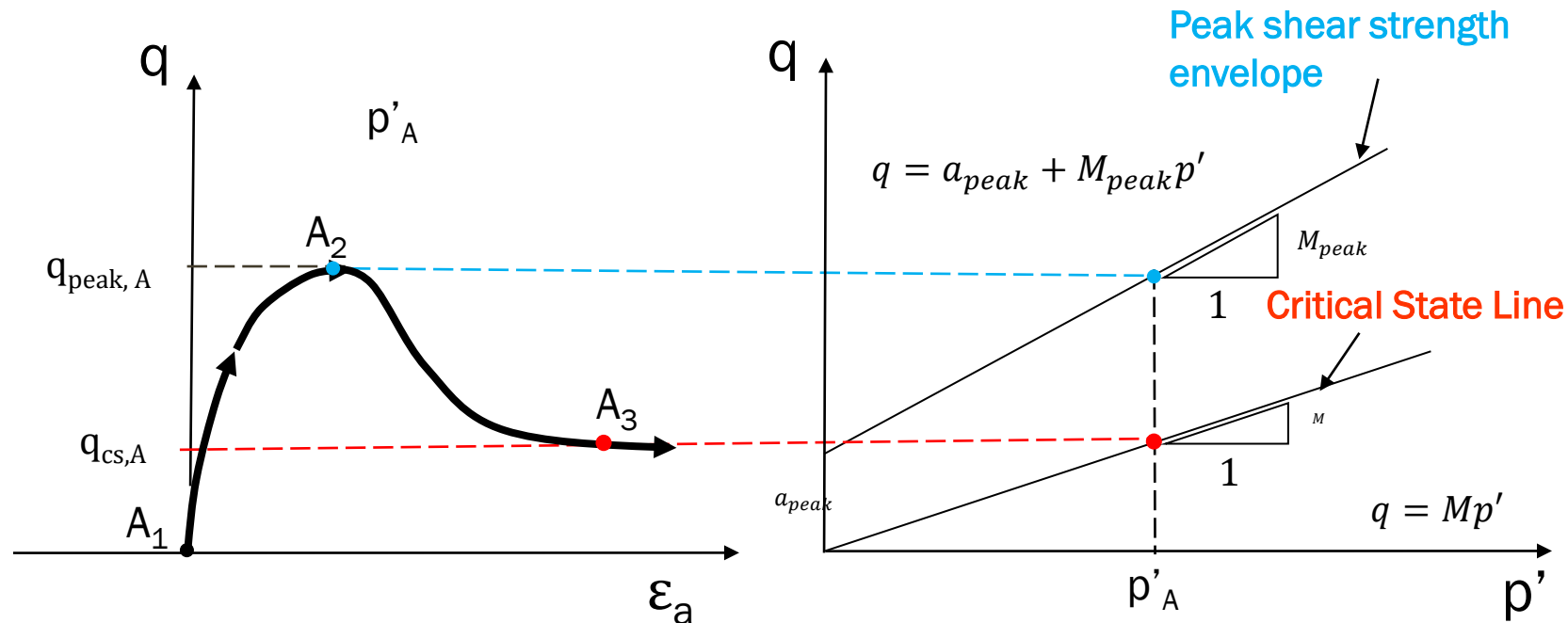
Two samples of the **same soil** consolidated at the **same  $p'$**  in **drained** conditions but **different OCRs**:

- A → NC       $q_{peak,A} = q_{cs,A}$
- B → OC       $q_{peak,B} \neq q_{cs,B}$



# Critical state and shear strength

Soil Strength concept : Mohr-Coulomb failure law

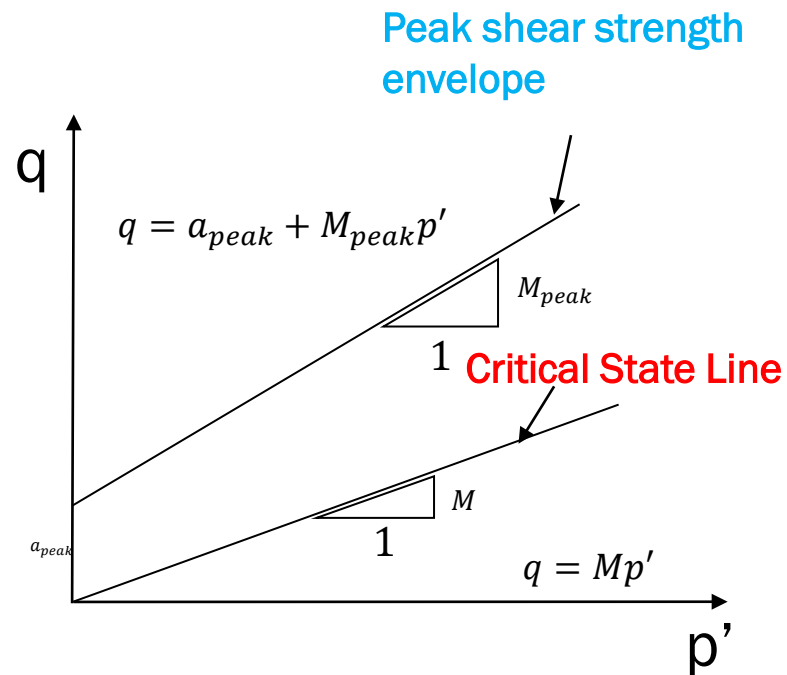


→ Soil strength determined through the Mohr-Coulomb failure law correspond to the peak strength

→ Soil strength definition should be selected based on the engineering problem of interest

# Critical state and shear strength

## Soil Strength concept : Mohr-Coulomb failure law



→ Different samples with the **same initial density** are sheared at **different confinement stresses**.  
The **peak values** are selected.

➔ NOT UNIQUE (initial void ratio dependency)

$$q = a_{peak} + M_{peak}p'$$

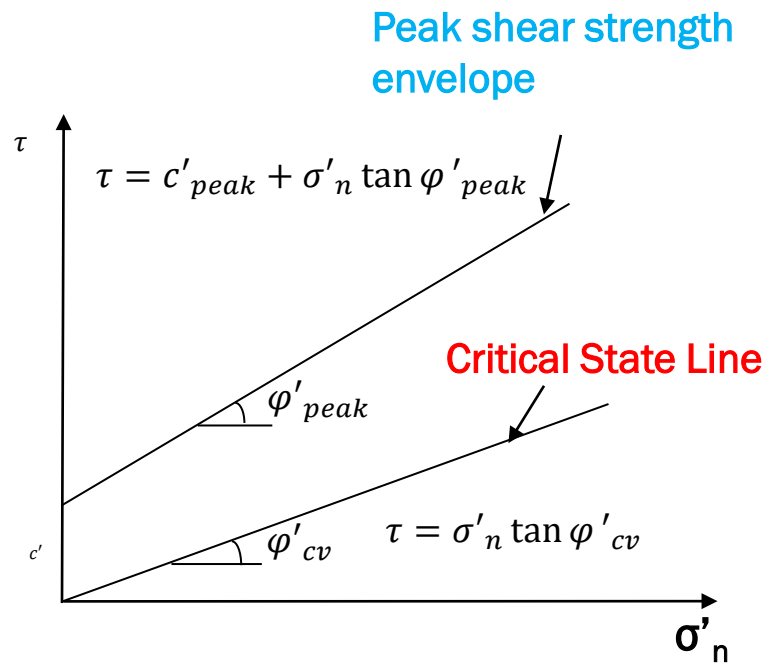
→ Different samples are sheared at **different confinement stresses**.  
The **critical values** are selected.

➔ UNIQUE (no initial void ratio dependency)

$$q = Mp' \quad M : \text{material parameter}$$

# Critical state and shear strength

## Soil Strength concept : Mohr-Coulomb failure law



→ From the q-p' plane to the  $\tau - \sigma'_n$  plane:

$$\tau = c'_{peak} + \sigma'_n \tan \varphi'_{peak} \left[ \begin{array}{l} \varphi'_{peak} = \arcsin \left( \frac{3M_{peak}}{6 + M_{peak}} \right) \\ c'_{peak} = \frac{a_{peak}}{M_{peak}} \tan \varphi'_{peak} \end{array} \right.$$

→ From the q-p' plane to the  $\tau - \sigma'_n$  plane:

$$\tau = \sigma'_n \tan \varphi'_{cv} \quad \varphi'_{cv} = \arcsin \left( \frac{3M}{6 + M} \right)$$

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- Experimental evidences Conventional Triaxial Compression tests
  - Isotropic drained compression
  - Shearing phase on a NC soil in drained and undrained conditions
  - Shearing phase on a OC soil in drained and undrained conditions
- Critical state and shear strength
- **Examples**

# Example

For a given soil, the critical state line is defined by the following soil parameters:

$$M = 0.9, \lambda = 0.25, \Gamma = 3.0$$

The soil was subjected to conventional drained and undrained triaxial compression tests, calculate the values of  $q$ ,  $p'$  and  $v$  at the critical state as well as the pore pressure for the undrained test when the soil initial condition is:

a)  $p'_i = 100 \text{ kPa}, q_i = 0 \text{ kPa}, v_i = 2.0$

b)  $p'_i = 100 \text{ kPa}, q_i = 0 \text{ kPa}, v_i = 1.7$

# Example (a) – Drained loading

Soil parameters:  $M = 0.9, \lambda = 0.25, \Gamma = 3.0$

a)  $p'_i = 100 \text{ kPa}, q_i = 0 \text{ kPa}, v_i = 2.0$

$$\Delta q = 3 \cdot \Delta p', \quad q_{cs} = M \cdot p'_{cs}$$

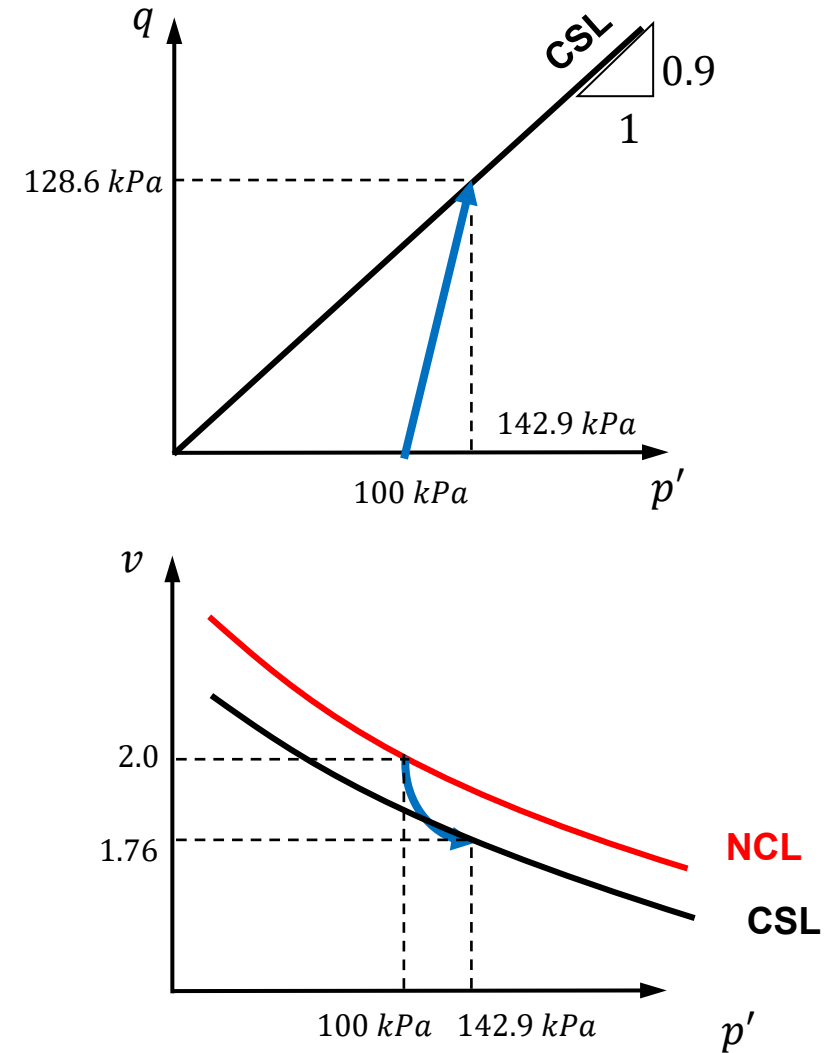
Drained loading  $\rightarrow$  ESP = TSP  $\rightarrow \Delta q = q_{cs} = 0.9 \cdot p'_{cs} = 3 \cdot \Delta p'$

$$p'_{cs} = p'_i + \Delta p'$$

$$0.9p'_i + 0.9\Delta p' = 3\Delta p' \rightarrow \Delta p' = 42.9 \text{ kPa}$$

$$p'_{cs} = 100 + 42.9 = 142.9 \text{ kPa}, \quad q_{cs} = 0.9 \cdot 142.9 = 128.6 \text{ kPa}$$

$$v_{cs} = \Gamma - \lambda \cdot \ln(p'_{cs}) = 3.0 - 0.25 \cdot \ln(142.9) = 1.76$$



# Example (a) – Undrained loading

Soil parameters:  $M = 0.9, \lambda = 0.25, \Gamma = 3.0$

a)  $p'_i = 100 \text{ kPa}, q_i = 0 \text{ kPa}, v_i = 2.0$

$$\Delta q = 3 \cdot \Delta p', \quad q_{CS} = M \cdot p'_{CS}$$

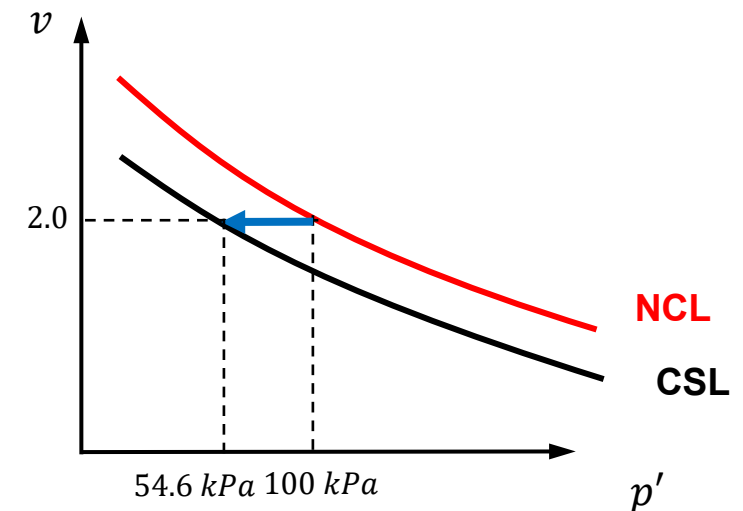
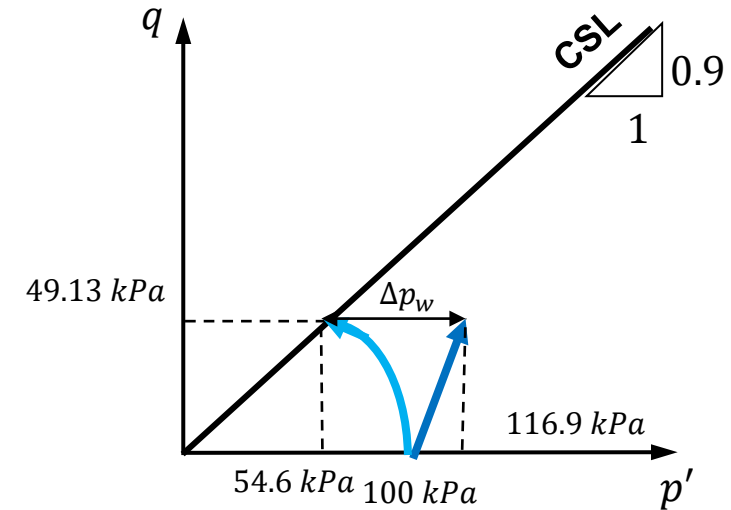
Undrained  $\rightarrow \varepsilon_v = 0 \rightarrow v_{CS} = v_i = \Gamma - \lambda \cdot \ln(p'_{CS})$

$$p'_{CS} = \exp\left(\frac{\Gamma - v}{\lambda}\right) = \exp(4) = 54.6 \text{ kPa}$$

$$q_{CS} = 0.9 \cdot 54.6 = 49.13 \text{ kPa}$$

$$\Delta q = q_{CS} = 3 \cdot \Delta p \rightarrow p = \Delta p + p_i = \frac{49.13}{3} + 100 = 116.4 \text{ kPa}$$

$$p_w = p - p' = 61.7 \text{ kPa}$$



# Example (b) – Drained loading

Soil parameters:  $M = 0.9, \lambda = 0.25, \Gamma = 3.0$

b)  $p'_i = 100 \text{ kPa}, q_i = 0 \text{ kPa}, v_i = 1.7$

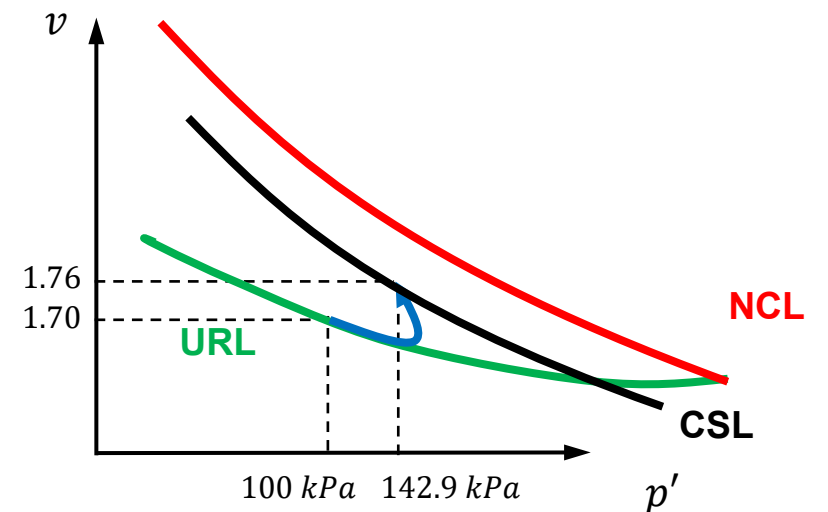
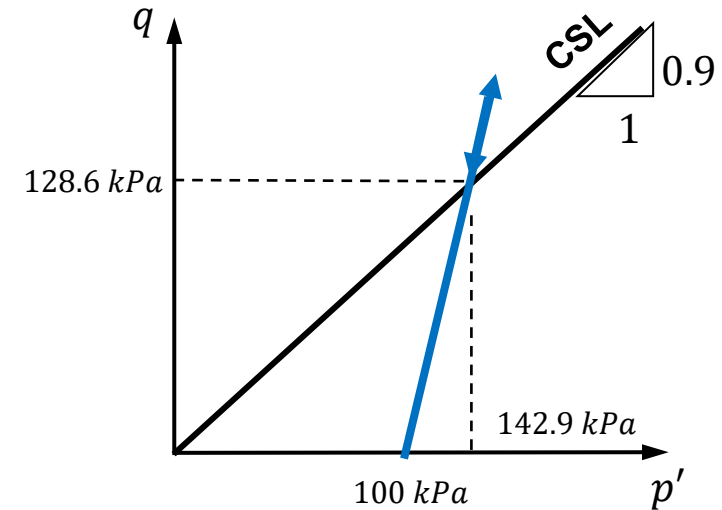
$$\Delta q = 3 \cdot \Delta p', \quad q_{cs} = M \cdot p'_{cs}$$

Drained loading  $\rightarrow$  ESP = TSP  $\rightarrow \Delta q = q_{cs} = 0.9 \cdot p'_{cs} = 3 \cdot \Delta p'$

$$p'_{cs} = p'_i + \Delta p' \rightarrow 0.9p'_i + 0.9\Delta p' = 3\Delta p' \rightarrow \Delta p' = 42.9 \text{ kPa}$$

$$p'_{cs} = 142.9 \text{ kPa}, \quad q_{cs} = 128.6 \text{ kPa}$$

$$v_{cs} = \Gamma - \lambda \cdot \ln(p'_{cs}) = 1.76$$



# Example (b) – Undrained loading

Soil parameters:  $M = 0.9, \lambda = 0.25, \Gamma = 3.0$

b)  $p'_i = 100 \text{ kPa}, q_i = 0 \text{ kPa}, v_i = 1.7$

$$\Delta q = 3 \cdot \Delta p', \quad q_{CS} = M \cdot p'_{CS}$$

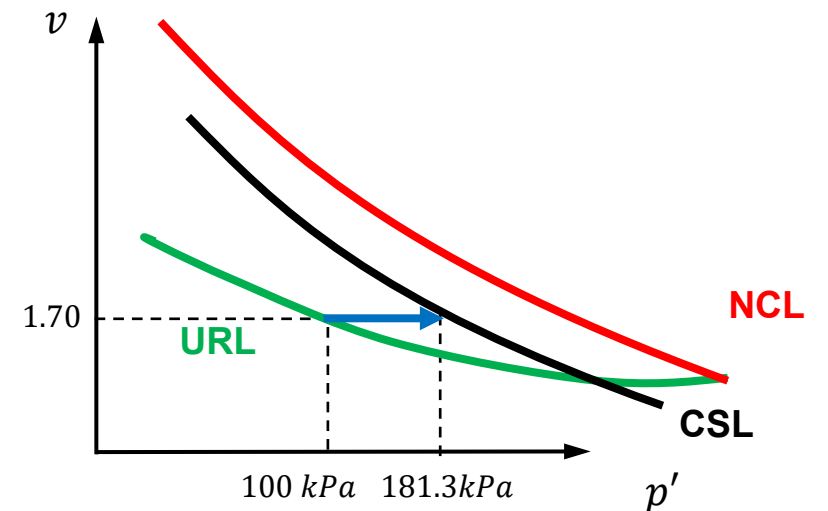
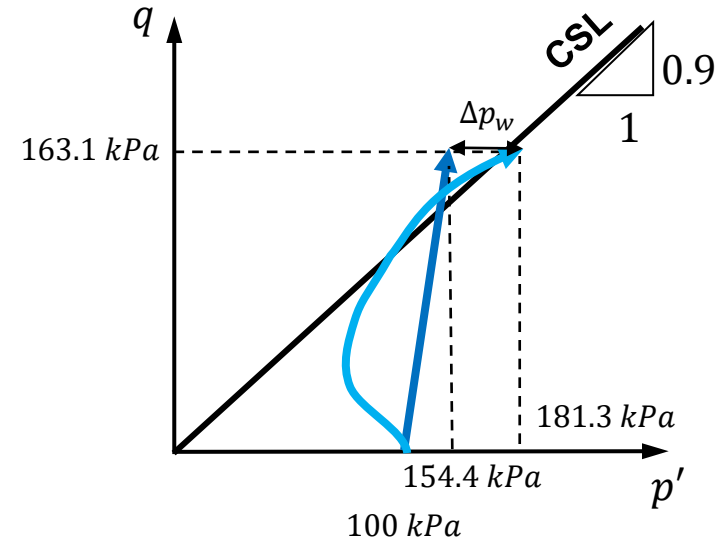
Undrained  $\rightarrow \varepsilon_v = 0 \rightarrow v_{CS} = v_i = \Gamma - \lambda \cdot \ln(p'_{CS})$

$$p'_{CS} = \exp\left(\frac{\Gamma - v}{\lambda}\right) = \exp(5.2) = 181.3 \text{ kPa}$$

$$q_{CS} = 0.9 \cdot 181.3 = 163.14 \text{ kPa}$$

$$\Delta q = q_{CS} = 3 \cdot \Delta p \rightarrow p = \Delta p + p_i = \frac{163.14}{3} + 100 = 154.38 \text{ kPa},$$

$$p_w = p - p' = -26.9 \text{ kPa}$$



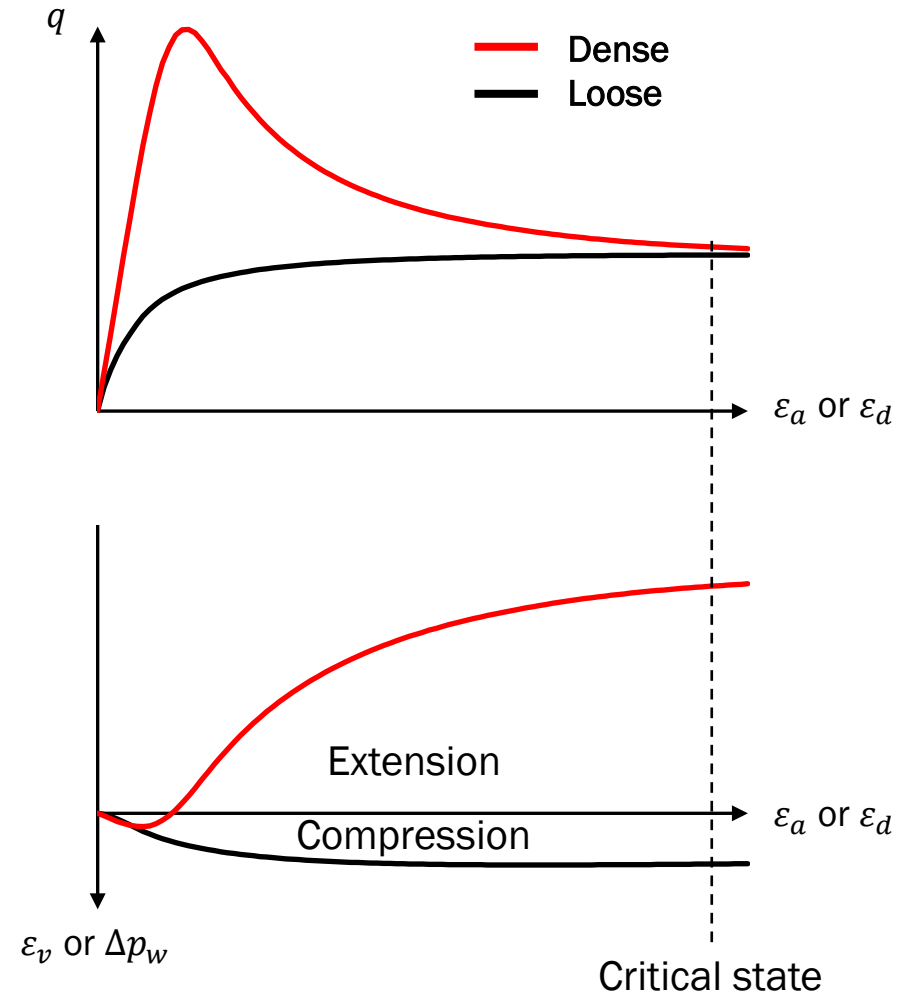
# Summary

## Critical state:

Ultimate state in which shearing can continue without:

- Any change in volume
- Any change in the mean effective stress
- Any change in the deviatoric stress

$$\frac{\partial \varepsilon_v}{\partial \varepsilon_d} = \frac{\partial p'}{\partial \varepsilon_d} = \frac{\partial q}{\partial \varepsilon_d} = 0$$



# Summary

- The critical state is a state in which the shearing can continue without any tendency of the soil to change its volume. It corresponds to reaching a constant volume condition in the case of a drained test, and to reaching a constant water pressure condition in the case of an undrained test.
- The critical state represents a lower bound to the strength of soils.
- Critical State Line (CSL) is defined in the  $q - p' - v$  space
- CSL does not depend on the test conditions (drained or undrained)
- CSL does not depend on the state of the geomaterial (NC or OC)
- The critical state parameters are material parameters of the tested geomaterial ( $\Gamma$ ,  $\lambda$  and  $M$ )
- The CSL of the  $q-p'$  plane can also be represented in the  $\tau-\sigma'_n$  plane (critical or ultimate shear strength envelope) and it corresponds to a Mohr-Coulomb failure law with  $c'=0$  and  $\varphi'_{cv}$

Thank you for your attention

